ON THE SUPPLY AND DEMAND FOR ENERGY

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THESIS

ON THE SUPPLY AND DEMAND
FOR ENERGY

by

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June 1974

Thesis Advisor: David Whipple

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ON THE SUPPLY AND DEMAND FOR ENERGY		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis June 1974
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Fariborz Golshani Javadi		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE June 1974
		13. NUMBER OF PAGES 83
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Conjunctive management of energy resources; Analytical Model for management of energy resources; Supply and Demand for Energy; Empirical and Analytical Study of Supply and Demand for Energy; Energy Management.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This document begins with a review of the world energy consumption, it is shown that there is a strong relationship between economic growth and energy consumption. This serves as background for the development of a comprehensive analytical model capable of quantitively evaluating the impact of energy related decisions. The general model developed is descriptive of exploration, extraction, storage, import and processing of

Block 20 - ABSTRACT (Cont.)

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DD Form 1473 (BACK)
1 Jan 73
S/N 0102-014-6601

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Submitted in partial fulfillment of the
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MASTER OF SCIENCE IN MANAGEMENT

from the

NAVAL POSTGRADUATE SCHOOL
June 1974

ABSTRACT

This document begins with a review of the world energy consumption, it is shown that there is a strong relationship between economic growth and energy consumption. This serves as background for the development of a comprehensive analytical model capable of quantitively evaluating the impact of energy related decisions. The general model developed is descriptive of exploration, extraction, storage, import and processing of energy resources. The analytical model also takes into account the relationship between these aspects of energy production and the market prices of energy resources.

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I. INTRODUCTION

One of the most important issues in the world today is the soaring prices of energy resources, this has caused considerable anxiety in the energy consuming nations concerning their ability to maintain economic growth and financial stability.

For example, in his second economic report, President Nixon cautioned that:

> "The new problems we face are of such enormity that there may be a temptation to delay further progress toward trade and monetary reform. The drastic increase in oil prices will have a significant impact on both the domestic economies of all nations and international economic relationships.[1]

According to United Nations Statistical Office, the total energy consumption in the world increased by 4% in 1971. The world consumption of electricity increased by 6% over previous year; world production of crude petroleum increased by 5.5%; the world exports of natural gas increased by 36% over 1970. The nuclear power accounted for 2% of total energy produced, which was an increase of 38% over 1970. On the other side the use of solid fuels declined by 9% of total world energy consumption. [2]

Government revenues in oil producing nations have increased considerably due to the recent increases in the price of crude oil. In fact the posted price of Persian Gulf light crude oil was increased from U.S. $1.80 per barrel on June 1, 1973 to U.S. $11.50 per barrel on January 1, 1974 by OPEC

(organization of Petroleum Exporting Countries), consequently producing nations are accumulating more reserves and their surplus could rise from $10 billion in 1973 to $60 billion in 1974. [3]

On the other side, consuming nations, due to the higher oil prices would have to have current account deficits. In their conference in Washington the consuming nations agreed that:

> "...the present situation, if continued could lead to a serious deterioration in income and employment, intensify inflationary pressures, and endanger the welfare of nations. And a serious setback to the prospect for economic development of developing countries will result from their additional energy costs at current oil prices." [4]

The production and consumption of energy has significant impact on the economy and financial position of a country. Hence in the last few years, many studies have been conducted to evaluate the impacts of policy options such as price regulations, import regulations, environmental controls, etc. and several of these studies have used advanced analytical techniques.

In order to be able to trace the impact of policy changes and to be able to develop an analytical model for the conjunctive management of the energy sector of economy, it is essential to have an understanding of some of the complex interrelationships in the broad energy, demand and pricing situation.

This is the primary objective of this study. By conducting a conprehensive analysis of the world's energy consumption, energy-GNP relationship, and the existing literature,

an analytical model has been developed. The usefulness of this study for future efforts, in analyzing the consequences of alternative energy policy options, is covered in the conclusion.

II. AN EMPIRICAL STUDY OF THE ENERGY-GNP RELATIONSHIP

A. INTRODUCTION

> "Economic history attests to the crucial role played by the consumption of energy in advancing the material well-being of mankind. The industrial revolution and the growth of industry in the nineteenth century are almost synonymous with the significant contribution of coal to the development of the iron and steel industry, to railways, and to factory mechanization. In the twentieth century, electrification and motorized transport served to modify the process and many ways helped economic progress. 'High-energy civilization' has by now become a commonplace characterization of economically-advanced 20th Century society. The growth of population and the levels of prosperity achieved have depended upon, and resulted in (among other critical factors) the consumption of energy in large amounts. To the extent it materializes, continued demographic and economic growth will undoubtedly involve the steadily rising use of energy." [5]

It therefore becomes important to give particular attention to this relationship. At a minimum, the topic calls for an interlinking of physical energy data and of the nations' incomes and products (GNP) accounts.

Such a study could yield a number of important results:

- A better understanding of the behavior of energy consumption relative to gross national product.
- It would attempt to answer the question: On the average what levels of energy consumption are required for given levels of GNP. [6]

B. DEFINITION OF THE MODELS

We now turn to the specification of a basic model relating GNP and energy consumption.

In the first model the assumption is made, that a linear relationship exists between per capita energy consumption and GNP per capita, then:

(1) $E = \alpha+\beta G+\varepsilon$

where; E = energy per capita, (Dependent variable)

G = GNP per capita, (Independent variable)

α = constant

β = the coefficient representing the propensity to consume energy, (in physical units)

ε = stochastic disturbance.

Next the assumption is made, that an exponential relationship or logarithmic relation exists between these variables, then:

(2) $E = \bar{\gamma} G^{\delta} \bar{\varepsilon}$, or

(3) $\ln E = \ln\bar{\gamma} + \delta \ln G + \ln\bar{\varepsilon}$

where; $\gamma = \ln\bar{\gamma}$ = constant.

δ = coefficient representing the elasticity of energy consumption with respect to GNP

$\varepsilon = \ln\bar{\varepsilon}$ = stochastic disturbance

The models represented by equations (1) and (3) were applied for a number of samples of data corresponding to OECD Countries, (Organization for Economic Cooperation and Development). The results of the regression analysis are presented in section D, and the summary of principal statistical results appear in Table I.

C. DESCRIPTION OF DATA

The data used here are collected from United Nations Statistical yearbooks and Statistics of Energy for member countries of OECD listed in Appendix C. The selection of data was made because of the special nature of the problem which requires detailed statistics of GNP in constant prices, with corresponding values for population and energy consumption. The only available data with the appropriate characteristics were those of OECD countries and only for selected years, the specific data which is used is described in following lines.

1. Population

Data are taken from OECD Main Economic Indicators, and is measured in million, for years 1960, 1965, 1966, 1967, 1968, 1969, 1970 and 1971, total of 174 observations (22 countries multiplied by 8 years minus two observations for years 1966, 1967 of Australia, not being used, because of lack of corresponding values for energy consumption per capita during these two years.)

2. Energy Consumption per Capita

Data are taken from United Nations Statistical yearbooks; 1960, 1965, 1966, 1967, 1968, 1969, 1970 and 1971. It is measured in kilograms coal equivalent per capita. There are 174 observations.

3. Gross National Product

Data are taken from OECD Main Economic Indicators. GNP is measured in million U.S. dollars, at 1963 prices and

1963 exchange rates. The GNP figures are next divided by corresponding values of population to determine the GNP/capita, for the years 1960, 1966, 1967, 1968, 1969, 1970 and 1971 with a total of 174 observations.

4. By Sources of Energy

All data are taken from "Statistics of OECD countries for Energy, 1954-1968, Paris 1970" for the years 1966, 1967 and 1968, and divided by corresponding population figures (Appendix C).

a. Crude Petroleum

This is measured in thousands metric tons with a total of 60 observations (20 countries for the three years, 1966, 1967 and 1968).

b. Motor Gasoline

This is measured in million metric tons with a total of 63 observations (21 countries for the three years, 1966, 1967 and 1968).

c. Aviation Fuel

This is determined by sum of the aviation gasoline and jet fuel. It is measured in million metric tons with a total of 63 observations (21 countries for the three years, 1966, 1967 and 1968).

d. Hard Coal

This is measured in million metric tons with a total of 63 observations (21 countries for the three years, 1966, 1967 and 1968).

e. Kerosine

This is measured in million metric tons with a total of 63 observations (21 countries for the three years, 1966, 1967 and 1968).

f. Gas Oil

This is measured in million metric tons with a total of 63 observations (21 countries for the three years, 1966, 1967 and 1968).

g. Natural Gas

This is measured in billion cubic meters at 4200 KCal., with a total of 33 observations (11 countries for the three years, 1966, 1967 and 1968).

D. STATISTICAL RESULTS

For the application of the models a general-purpose statistical package, based on least-square method (SNAP/IEDA) has been used. The results of the analysis are as follows (with standard errors in parenthesis),

where; R^2 = Square of correlation coefficient.

$\overline{R}^2$ = Adjusted R^2.

F = The ratio of the variance of the residuals of the dependent variable before regression and the variance of the residuals of that variable after the regression.

NOBS = Number of observations.

M = The mean of variables.

K = Elasticity of consumption. When the linear model is used $K = \frac{\partial E}{\partial G} \frac{MG}{ME}$, where $\frac{\partial E}{\partial G}$ is the regression coefficient, MG and ME are the means of the independent variable and dependent variable respectively. When the constant elasticity model is used, K equals the coefficient δ.

1. Energy/Capita and GNP/Capita

a. Linear Model (Fig. 1)

$$E = -584.905 + \underset{(.093)}{2.625G}$$

$R^2 = .823$, $F = 797.1$, NOBS = 174,

ME = 3722.8621, MG = 1641.6658,

$$K = \frac{\partial E}{\partial G} \cdot \frac{MG}{ME} = 2.625 \times \frac{1641.6658}{3722.8621} = 2.625 \times .441$$

$K = 1.158$, $\overline{R}^2 = .822$

b. Constant Elasticity Model (Fig. 2)

$$\ln E = -0.902 + \underset{(.031)}{1.225} \ln G$$

$R^2 = .903$, $F = 1603.4$, NOBS = 174,

$K = 1.225$, $\overline{R}^2 = .902$

2. Crude Petroleum/Capita and GNP/Capita

a. Linear Model (Fig. 3)

$$Ep = 269.059 + \underset{(0.269)}{0.709G}$$

$R^2 = .107$, $F = 6.9$, NOBS = 60,

MEp = 1401.8915, MG = 1597.0630,

$$K = .709 \times \frac{1597.0630}{1401.8915}$$

$K = .709 \times 1.14 = .81$, $\overline{R}^2 = .092$

b. Constant Elasticity Model (Fig. 4)

$$\ln Ep = -0.334 + \underset{(.126)}{0.998} \ln G$$

$R^2 = .52$, $F = 62.8$, NOBS = 60

$K = .998$, $\bar{R}^2 = .488$

3. <u>Motor Gasoline/Capita and GNP/Capita</u>

a. Linear Model (Fig. 5)

$$Eg = -172.334 + \underset{(.021)}{0.258G}$$

$R^2 = .705$, $F = 145.9$, NOBS = 63,

$MEg = 244.5014$, $MG = 1616.1303$,

$$K = .258 \times \frac{1616.1303}{244.5014}$$

$K = .258 \times 6.610 = 1.705$, $\bar{R}^2 = .700$

b. Constant Elasticity Model (Fig. 6)

$$\ln Eg = -4.421 + \underset{(.061)}{1.321} \ln G$$

$R^2 = .885$, $F = 470.6$, NOBS = 63,

$K = 1.321$, $\bar{R}^2 = .883$

4. <u>Aviation Fuel/Capita and GNP/Capita</u>

a. Linear Model (Fig. 7)

$$Ea = -23.644 + \underset{(.006)}{0.045G}$$

$R^2 = .451$, $F = 50.1$, NOBS = 63,

$MEa = 48.4012$, $MG = 1616.1303$,

$$K = .045 \times \frac{1616.1303}{48.4012}$$

$K = .045 \times 33.390 = 1.503$, $\bar{R}^2 = .442$

b. Constant Elasticity Model (Fig. 8)

$$\ln Ea = -4.304 + \underset{(.166)}{1.061} \ln G$$

$R^2 = .402$, $F = 40.9$, NOBS = 63,

$K = 1.061$, $\overline{R}^2 = .392$

5. Hard Coal/Capita and GNP/Capita

a. Linear Model (Fig. 9)

$$Eh = 69.885 + \underset{(.123)}{0.470}G$$

$R^2 = .192$, $F = 14.5$, NOBS = 63,

MEh = 828.7752, MG = 1616.1303,

$$K = .470 \times \frac{1616.1303}{838.7752}$$

$K = .470 \times 1.95 = .92$, $\overline{R}^2 = .179$

b. Constant Elasticity Model (Fig. 10)

$$\ln Eh = -.917 + \underset{(.267)}{.957}G$$

$R^2 = .174$, $F = 12.9$, NOBS = 63,

$K = .957$, $\overline{R}^2 = .160$

6. Kerosine/Capita and GNP/Capita

a. Linear Model (Fig. 11)

$$Ek = 6.973 + \underset{(.005)}{.014}G$$

$R^2 = .121$, $F = 8.4$, NOBS = 63,

MEk = 29.6929, MG = 1616.1303,

$$K = .014 \times \frac{1616.1303}{29.6939}$$

$K = .014 \times 54.43 = .762$; $\overline{R}^2 = .107$

b. Constant Elasticity Model (Fig. 12)

$R^2 = .06$, $\overline{R}^2 = .045$

7. <u>Gas Oil/Capita and GNP/Capita</u>

a. Linear Model (Fig. 13)

$$Eo = 229.796 + \underset{(.065)}{.452G}$$

$R^2 = .443$, $F = 48.6$, $NOBS = 63$,

$MEo = 960.6031$, $MG = 1616.1303$

$$K = .452 \times \frac{1616.1303}{960.6031}$$

$K = .452 \times 1.682 = .760$, $\bar{R}^2 = .434$

b. Constant Elasticity Model (Fig. 14)

$$\ln Eo = -1.462 + \underset{(.071)}{1.121} \ln G$$

$R^2 = .805$, $F = 251.7$, $NOBS = 63$,

$K = 1.121$, $\bar{R}^2 = .802$

8. <u>Natural Gas/Capita and GNP/Capita</u>

a. Linear Model (Fig. 15)

$$En = -2013.059 + \underset{(.184)}{1.708G}$$

$R^2 = .736$, $F = 86.4$, $NOBS = 33$,

$MEn = 992.0987$, $MG = 1759.8145$,

$$K = 1.708 \times \frac{1759.8145}{992.0987}$$

$K = 1.708 \times 1.77 = 3.03$, $\bar{R}^2 = .727$

b. Constant Elasticity Model (Fig. 16)

$$\ln En = -27.099 + \underset{(.609)}{4.392} \ln G$$

$R^2 = .626$, $F = 52.0$, $NOBS = 33$,

$K = 4.392$, $\bar{R}^2 = .614$

The results of the empirical analysis are summarized in Table I.

TABLE I

SUMMARY OF THE RESULTS

MODEL		Energy Capita	Crude Petrol	Motor Gasoline	Aviation Fuel	Hard Coal	Kerosine	Gas Oil	Nat Gas
Linear	R^2	.823	.107	.705	.451	.192	.121	.443	.736
Cons Elas	R^2	.903	.520	.885	.402	.174	.06	.805	.626
Linear	$\bar{R}^2$	.822	.092	.700	.442	.179	.107	.434	.727
Cons Elas	$\bar{R}^2$	.902	.488	.883	.392	.160	.045	.802	.614
Linear	α	-585.905	269.059	-172.334	-23.644	69.885	6.973	229.796	-2013.059
Cons Elas	γ	- .902	- 0.334	- 4.421	- 4.304	- .917	----	- 1.462	- 27.099
Linear	β	2.625 (.093)	.709 (.269)	.258 (.021)	.045 (.006)	.470 (.123)	.014 (.005)	.452 (.065)	1.708 (.184)
Cons Elas	δ	1.225 (.031)	.998 (.126)	1.321 (.061)	1.061 (.166)	.957 (.267)	----	1.121 (.071)	4.392 (.609)
Linear	K	1.158	.81	1.705	1.503	.92	.762	.760	3.03
Cons Elas	K	1.225	.998	1.321	1.061	.957	----	1.121	4.392
Linear	F	797.1	6.9	145.9	50.1	14.5	8.4	48.6	86.4
Cons Elas	F	1603.4	62.8	470.6	40.9	12.9	----	251.7	52.0
NOBS		174	60	63	63	63	63	63	33

E. INTERPRETATION OF THE RESULTS

By referring to the summary of results presented by Table I, we can reach the following conclusions:

1) The logarithmic model was a better fit to the data on energy per capita, crude petroleum, gasoline, and gas oil. While the simple linear model was a better fit to data on aviation fuel, hard coal, kerosine and natural gas.

2) There is a higher correlation between GNP and aggregate energy consumption than between GNP and different energy sources. This could be explained by the fact that each member country of the OECD consumes different sources of energy as the primary mixture for energy production. There are a number of economic and technological factors affecting the mix of the primary energy sources within a country. It is therefore necessary to pay particular attention to the special factors underlying the historic interfuel composition of total energy consumption and to the policy tools available to alter the composition.

III. <u>THE ANALYTICAL STUDY OF ENERGY PRODUCTION AND MANAGEMENT</u>

A. INTRODUCTION

There are considerable efforts taking place in the private and public sectors of the economics profession in the recent years, to satisfy the need for a quantitative economic theory of energy conservation and management. Such efforts provide a point of departure for the development of analytical models which may be usefule in projecting future energy requirements and the short-term, and the long-term adequacy of supplies to meet them.

The energy situation comprises a wide range of interrelated socio-economic and institutional factors. These include:

- Balance of payments.
- National security and stockpiling.
- Environmental costs and user costs.
- Protection of energy related industries.
- Resource production (exploration, development, extraction, processing, refining) and relative facilities.
- Transportation (means, modes and facilities).
- Marketing and Distribution systems.
- Substitutabilities among different energy sources.
- Development of new sources of energy.

- Problem involving risks and uncertainties in terms of domestic and foreign investments in exploration as well as for import supplies, prices, and uncertainties related to technologies.
- Interdependencies among the stages of activity and the policy alternatives.
- Conjunctive management at national, regional, and state level versus an individual firm or a single industry.

The purpose of this chapter is to present a summary of the literature concerning the production and management of the natural resources.

B. RESULTS OF THE ECONOMIC THEORY OF EXHAUSTIBLE RESOURCES

The theory of exhaustion considers the losses firms may incure if they do not consider the resource availability in future generations.

Results indicate that different structure of industry may produce different output patterns, for example private ownership may lead to decisions different from those of centralized management. In addition, the time-path of exploitation, when the costs are affected by cumulative production is different from conditions where the increasing costs due to cumulative output are ignored.

The general format of the models presented under Exhaustion theory consist of maximization of present value of future profits subject to limited supply of resources. In

following section simple examples of these models have been presented.

C. SIMPLE CASES IN THEORY OF EXHAUSTION

1. Case (1)

Considering the case where a private distributor of fuel is hoarding a fixed quantity of it for a variable length of time without occurring additional costs, and the selling price of fuel increases by a known function $P = p(t)$; as time goes on. What would be the optimum time for him to sell stored supply knowing that he can sell all quantity Q available? [7]

It is assumed that interest is added continuously at a constant rate of r%.

If (y) denotes the present value of the fuel sold after (t) years, then

(1) $y = Q \cdot P(t) \cdot e^{-rt}$

The condition for optimum time are given by:

$$\frac{dy}{dt} = 0 \;, \quad \frac{d^2y}{dt^2} < 0$$

Knowing that the log y is a "monotonic transformation" of y. (This means that whenever y is increasing log y is also increasing, and whenever y is falling its logarithm is also falling.) Therefore, the point corresponding to the maximum of y is also maximum of log y. Then

(2) $\log y = \log P(t) + \log Q - rt$, or

(3) $\log y = F(t) + \log Q - rt$

where $F(t) = \log P(t)$. Then:

$$\frac{d}{dt}(\log y) = \frac{1}{y}\frac{dy}{dt} = F'(t)-r$$

$$\frac{d^2}{dt^2}(\log y) = \frac{d}{dt}(\frac{1}{y}\frac{dy}{dt}) = \frac{1}{y}\frac{d^2y}{dt^2} - \frac{1}{y^2}(\frac{dy}{dt})^2 = F''(t)$$

The condition for optimum selling time are thus

$$F''(t) < 0 \quad \text{and} \quad F'(t) = r$$

From Eq. (3) the first condition is that

$$F'(t) = \frac{d}{dt}(\log P(t)) = \frac{P'(t)}{P(t)} = r$$

Where $\frac{P'(t)}{P(t)}$ determines the rate of growth[1] of selling price at period (t), is equal to the constant rate of market interest r. By the second condition

$$F''(t) = \frac{d^2}{dt^2}(\log P(t)) = \frac{d}{dt}\{\frac{P'(t)}{P(t)}\} < 0$$

which implies the rate of growth for selling price must be decreasing at the optimum time.

Result: If the rate of increase for selling price is constant and equal to the market rate of interest over the time; then owner would be indifferent whether he sells his reserves at price P_0 now or at a price $P=P_0e^{rt}$ after time t. This could be shown from Eq. (1). The present value would be: $y = QPe^{-rt}$

where $P=P_0e^{rt}$ implies

(4) $y = QP_0e^{rt} \cdot e^{-rt} = QP_0$

---

[1] Appendix A, Section 1.

2. Case (2)

Now considering the case under free competition when the rate of increase for selling price is equal to the market rate of interest. The formula $P=P_0e^{rt}$ fixes the relative prices at different times and the owner would be indifferent about a unit of fuel being sold now or in the future. (P being interpreted as the net price received after paying the costs of extraction and placing upon the market.) [8] And Q being the total amount of supply available, and the quantity q distributed at any given time is a continuous function of price and time, if t=T being the time of final exhaustion, it could be written:

(5) $\quad q = q(P,t) \quad$ when $t=0$, $P=P_0$; then $q=q_0$.

and when $t=T$, $P=P_0e^{rT}$ and $q=0$. Then

(6) $$Q = \int_0^T q(P,t)dt = \int_0^T q_0(P_0e^{rt},t)dt$$

The question will be the determination of t=T the time of final exhaustion to maximize the present value of his revenue. Since at t=T, q=0 it implies:

(7) $$q = q(P,t) = q(P_0e^{rT},T) = 0$$

and consequently

$$y = \int_0^T q(P,t)\cdot P\cdot e^{-rt}dt = \int_0^T q(P,t)\cdot Pe^{rt}\cdot e^{-rt}dt$$

$$= \int_0^T q(P,t)\cdot P_0dt = P_0 \int_0^T q(p,t)dt$$

From Eq. (6) follows $y=P_0Q$ where P_0 and Q are both constant.

Result: In the case of free competition, when also the rate of price increase is constant and equal to market rate of interest, the owner is indifferent to q quantity supplied at any time t from Q his total reserve and the T final time of exhaustion.

D. A GENERAL CASE IN THEORY OF EXHAUSTION

Considering the case under monopoly when a certain commodity is available in fixed supply Q with the rate of production $q=q(t)$ over a time period from t_0 to t_2 [9, 10]. Production then is subject to the constraint.

$$(8) \qquad \int_{t_0}^{t_2} q(t)dt \leq Q$$

The firm would plan to maximize the present value of its profits

$$(9) \qquad y = \int_{t_0}^{t_2} \Pi[q(t),t]\cdot e^{-rt}dt$$

where y is net present value; Π total profits; $q(t)$ output; r the continuous interest rate.

The question involves the selection of the optimal times for starting and completing output and the optimal output pattern. The solution implies resolving the Lagrangean Equation.

$$(10) \qquad L = \int_{t_0}^{t_2} \Pi[q(t),t]\cdot e^{-rt}dt - \lambda \int_{t_0}^{t_2} q(t)dt-Q \ .$$

where λ is the Lagrangean multiplier; and Q total availability.

The optimal production pattern is determined by the Euler equation of the Calculus of Variation,[2] and the boundary conditions are the attaining stationary value at t_0 and t_2 (Eq. 10). The Euler equation requires that

$$\frac{\partial L}{\partial q} - \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

And because $\dot{q}=\frac{dq}{dt}$ does not appear in Eq. (10), it therefore does not effect profit, so the Euler equation reduces to $\frac{\partial L}{\partial q} = 0$. [11]

(11) $$\frac{\partial L}{\partial q} = e^{-rt} \int_{t_0}^{t_2} \frac{\partial \Pi}{\partial q} \frac{\partial q}{\partial t} dt - \lambda q = 0$$

By substitution of dq for $\frac{\partial q}{\partial t}dt$; Eq. (10) could be written as

(12) $$\frac{\partial L}{\partial q} = e^{-rt} \int_{t_0}^{t_2} \frac{\partial \Pi}{\partial q} dq - \lambda q \quad \text{or}$$

(13) $$\int_{t_0}^{t_2} \frac{\partial \Pi}{\partial q} dq = \lambda q e^{rt} \qquad \text{or}$$

(14) $$\frac{\partial \Pi}{\partial q} = \lambda e^{rt}$$

By introducing the concept of marginal profit, the revenue (R) minus Cost (C) would determine the profit. Using letters P for price; AC, average costs; ϕ, average profits; MR = $\frac{\partial R}{\partial q}$, marginal revenue; MC = $\frac{\partial C}{\partial q}$, marginal cost; and M = $\frac{\partial \Pi}{\partial q}$, marginal profit. Then Eq. (14) can be written as

(15) $$M\Pi(t) = \frac{\partial \Pi}{\partial q} = MR - MC = \lambda e^{rt}$$

The conventional second order equations $\frac{\partial^2 \Pi}{\partial q^2} < 0$ and $\frac{\partial^2 R}{\partial q^2} < \frac{\partial^2 C}{\partial q^2}$ also apply.

---

[2] Appendix B.

In fact $\Pi = R - C = P(q)\cdot q - C(q)$,

$$M\Pi = \frac{\partial \Pi}{\partial q} = \frac{\partial R}{\partial q} - \frac{\partial C}{\partial q} = P(q) + q\cdot P(q) - C(q) \text{ and,}$$

(16) $$\frac{\partial^2 \Pi}{\partial q^2} = \frac{\partial^2 R}{\partial q^2} - \frac{\partial^2 C}{\partial q^2} = P'(q) + P'(q) + q\cdot P''(q) - C''(q)$$

where in general $P(q)$ being the demand curve implies its slope $P'(q) < 0$ and $P''(q) \geq 0$. Considering increasing cost industry, it then implies $C''(q) > P''(q) > 0$; consequently making Eq. (16) negative or

$$\frac{\partial^2 \Pi}{\partial q^2} < 0, \text{ or } \frac{\partial^2 R}{\partial q^2} - \frac{\partial^2 C}{\partial q^2} < 0, \text{ or } \frac{\partial^2 R}{\partial q^2} < \frac{\partial^2 C}{\partial q^2}.$$

Under free competition, $P \equiv MR$ so Eq. (15) becomes

(17) $$P - MC = \lambda e^{rt}$$

and with constant cost, this reduces further to

(18) $$P - AC = \lambda e^{rt}$$

Result: From Eq. (15) it derives:

(19) $$\Pi = \lambda q(t) e^{rt}$$

dividing it by $q(t)$ results:

(20) $$\phi(t) = \lambda e^{rt}$$

since Eq. (15) also hold at time t_2, the optimum production path is where marginal profit equals average profit. Since

$$\Pi(t) = \phi(t)\cdot q(t), \text{ then } M\Pi(t) = \phi(t) + \left(\frac{\partial \phi}{\partial q}\right)q(t).$$

The equality $M\Pi(t) = \phi(t)$ implies $q\left(\frac{\partial \phi}{\partial q}\right) = 0$ which occurs if q or $\frac{\partial \phi}{\partial q}$ is zero; the firm either produces nothing at t_2 or produces the output that maximizes the average profits at t_2.

E. RESULTS OF ECONOMIC MODELS OF PRODUCTION IN RESOURCE INDUSTRIES

The theory of exhaustion has been developed without explicitly considering the role of investment associated with resources, the interaction among production rates, investment rates, technological changes and exploration expenditures.

In recent studies efforts have taken place to develop economic models to include these factors. According to these studies, the stages of activity necessary for the production of energy commodities involve numerous interrelated aspects. For example, the optimal extraction rate of a given resource and associated investments in capital stock are related to exploration and importation policies of the resources as well as the substitutability among the resources. [12]

The supply of particular resource should be obtained from domestic production and imports until marginal import costs equal the marginal costs of extracting domestic stocks, and be allocated between current use and stockpiles for future consumption, until the marginal contribution of resource to the current use equals its marginal contribution to future use. [13]

By considering the user costs defined by A. Scott as "the present value of future profits foregone by a decision to produce a unit of output today." The use of imports in current production would reduce the rates of domestic extraction

and consequently the user cost, or imports could be used to increase the stockpiles to reduce the future scarcity of the resource. [14]

The above brings to attention the costs which accompany the import restrictions and/or stock controls.

The equality of the user costs specially among substitutable resources are of special interest in the conjunctive management of energy-related resources.

An important question arises in terms of the impact of subsidies, depletion allowances granted to one resource, on the optimal rates of exploration expenditures, importation and production of other resources. [15]

Of particular attention are the social attitudes towards the generation of pollutants, and the inclusion of pollution abatement costs in the processing costs and production costs, which may be imposed in terms of social costs.

Considering the problems associated with petroleum production and reservoir management, the results suggest that the amount of pressure which exists in the reservoir, and the time-path of pressure depletion, are two influential factors on the production rate and the amount of oil recoverable [16]

It should be noted that in many cases pressure may be maintained or increased artificially by incurring additional expenditures, which implies that the recoverable stock, as well as production rates, generally will depend strongly on investment. This dependence is not merely on a cumulative

investment, but also on the time-path of investment. Thus the impact of rapid production rates, in the face that the recoverable stock is rate dependent, may be offset to some extent by investments. [17]

IV. DEVELOPMENT OF AN ANALYTICAL MODEL

A. INTRODUCTION

Because of increasing concern over the management of "Energy Sector" of economy as whole, a comprehensive analytical model is required, to be able to translate and quantitatively evaluate the impact of energy related procedures.

In the three preceding chapters a brief review of literature, past and current works was performed, to give an understanding of some of the complex interrelationships in this sector.

In specifying the scope and coverages of this model, a number of requirements, could be identified, to be covered by the comprehensive model. These requirements could be listed as follows: [18]

- The model should cover all energy forms.
- The model should consider supply and demand interactions or the mechanism for balancing supply and demand.
- The model should take into account interfuel substitution.
- The model must take into consideration the impact of price and other variables on the demand and supply of the specific fuels.
- The model must consider the impact of technological changes on both the supply and demand of fuels and energy.

- The model must be disaggregated at the regional level.
- The model must be able to use the available data for future analyses.

Note that the environmental consequences have been left out in this study.

The economic model prepared by R. Cummings and David Whipple [19], is an initial attempt at a comprehensive model of the discovery, extraction, importation, and stockpiling of primary energy resources, integrating the production of final energy commodities, it is believed its further expansion and refinements, will make it suitable for use in constructing an analytical model, presented in next section, feasible to the conjunctive management problems of energy sector, by covering the above mentioned requirements.

B. THE MODEL

By assuming the existence of "j" resource fields in which any of "R" resources are extracted from domestic deposits and/or stored in stockpiles. For each field j=1,2,...,J; the intertemporal changes in stocks of domestic deposits of the resource r, r=1,2,...,R, are described by the following:

(1) $X_{rj}^{t+1} = X_{rj}^{t} - W_{rj}^{t}$, X_{rj}^{0} given, for all r=1,...,R; j=1,...,J; t=1,2,...T.

By (1), the sotck of resource r in field j at the end of period t, "X_{rj}^{t+1}" equals stocks at the beginning of the period "X_{rj}^{t}" minus extraction of r during t, "W_{rj}^{t}" which is a function of:

(2) $W^t_{rj} = \phi_{wj}(\Pi^t_{rj}, \rho^t_{rj}, Q^t_r)$,

where; Π^t_{rj} = The excess of "P_r"; the market price of resource r, over "C^{wt}_{rj}" the extraction cost of resource r in field j, during period t. [20]

ρ^t_{rj} = The excess of P_r over "C^{mt}_{rj}" the importation cost of resource r in field j, during period t. [21]

Q^t_r = The quantity of resource r, demanded in period t, which is a function of P_r; r=1,...,R the market prices of all resources during the period t.

(3) $Q^t_r = \phi_q(P_1, P_2, \ldots., P_R)$.

It is alos assumed, that periodic extractions of r in field j are subject to an upper bound of the form.

(4) $W^t_{rj} \leq g^t_{rj}(X^t_{rj}, K^t_j)$; $\frac{\partial g}{\partial X}, \frac{\partial g}{\partial K} \geq 0$ for all r, j and t.

The upper bound g^t_{rj} is intended to reflect the impacts of resource and capital stocks in terms of impinging on periodic rates of extraction. For example, given a capital stock "K^t_j", (Pumps, Pipe size, etc.), the smaller the recoverable stock of petroleum in petroleum reservoir (and therefore, the lower may be the pressure for natural derive), the samller one would expect the maximal periodic rate of extraction.

For a given resource stock, smaller capital stocks imply smaller periodic rates of extraction.

Further, it is assumed that r extracted from j's deposits are used for increasing stockpiles of r.

The intertemporal changes in stockpiles of the resource r, are described by the following:

(5) $S^{t+1}_{rj} = S^{t}_{rj}+W^{t}_{rj}+M^{t}_{rj}-\Delta^{t}_{rj}$, S^{0}_{rj} given, for all r, j and t.

By (5), stockpiles of r, in field j at the end of period t "S^{t+1}_{rj}" equals initial stockpiles, "S^{t}_{rj}" plus extraction "W^{t}_{rj}", plus imports "M^{t}_{rj}", minus substractions from stockpiles "Δ^{t}_{rj}" for current consumptions.

The following functional forms are assumed for Δ^{t}_{rj} and M^{t}_{rj}.

(6) $\Delta^{t}_{rj} = \phi_{\delta j}(\Pi^{t}_{rj=1,...,J}, \rho^{t}_{rj=1,...,J}, \mu^{t}_{rj=1,...,J}, Q^{t}_{r})$,

with $\frac{\partial\phi_{\delta}}{\partial\Pi} \geq 0$, $\frac{\partial\phi_{\delta}}{\partial\rho} \geq 0$, $\frac{\partial\phi_{\delta}}{\partial\mu} \geq 0$, $\frac{\partial\phi_{\delta}}{\partial Q} \geq 0$

(7) $M^{t}_{rj} = \phi_{mj}(\Pi^{t}_{rj=1,...,J}, \rho^{t}_{rj=1,...,J}, \mu^{t}_{rj=1,...,J}, Q^{t}_{r})$

where; μ^{t}_{rj} = The excess of P_r, over "C^{st}_{rj}" the storage cost of resource r in field j, during period t.

Stockpiles S^{t}_{rj} are assumed to be restricted by an upper bound N^{t}_{rj} which reflects the capital stocks and storage facilities. With $\overline{K}^{t}_{rj}$ defined as field's J capital stock for storage facilities at the beginning of period t, it is defined.

(8) $S^{t}_{rj} \leq N^{t}_{rj}(\overline{K}^{t}_{rj})$, $\frac{\partial N}{\partial\overline{K}} \geq 0$ for all r, j and t.

In the resource extraction system as characterized by (1)-(8), two observations are of interest at this juncture. First, statement (5) allows for the inclusion of port facilities for imports to be treated as "resource fields", i.e.,

for some $j=j^*$, $X^t_{rj^*}$, $W^t_{rj^*}$ are zero, and resource management issues in terms of "field" J^* concerns the management of stockpiles of r from imports. Second the transition equation for stocks given in (1) is in a simplified form for expository purposes, and does not allow for the dependence of recoverable stock on (earlier) time-rate of production and investment which may be of particular importance for resources such as petroleum. [22]

The system (1)-(8) is assumed to apply to fields which are known (for example, "proven" reserves at t=1) at the initial time-period.

To allow however, for exploration expenditures, e_{rz}, which may result in new fields, z-1,...,Z, with deposits of the resource.

Let exploration expenditures e_{rz} result in discovery of $h_{rz}(e_{rz})$ units of the resource r. (h, is essentially a measure of the "probably" rewards of exploration expenditures). By assuming a concave set of relationships of the form:

$$(9) \quad \sum_{t=1}^{T} h_{rz}(e^t_{rz}) \leq \overline{X}_{rz}, \text{ for all r, z, and}$$

$$\frac{\partial h_{rz}}{\partial e_{rz}} \geq 0, \ \frac{\partial h_{r1}}{\partial e_{r1}} > \frac{\partial h_{r2}}{\partial e_{r2}} > \ldots\ldots \frac{\partial h_{rZ}}{\partial e_{rZ}} .$$

By (9), initial exploration expenditures are for h_{r1} (which yields larger "Probable rewards" per unit of expenditure than h_{rz}, z=2,...,Z), until a bound $\overline{X}_{r1}$ is effective, after which $h_{r2},\ldots,h_{rZ}$ are applicable. The bounds $\overline{X}_{rz}$ are included to allow use of a continuous model as well as to allow for

establishment of new fields, (with reserves $\overline{X}_{rz}$) as a result of exploration expenditures. It is also assumed e^t_{rz} to have the form.

(10) $e^t_{rz} = \phi_e(\Pi_{rz}, \rho_{rz}, \eta_{rz}, Q_r)$,

with $\frac{\partial\phi_e}{\partial\Pi} \geq 0$, $\frac{\partial\phi_e}{\partial\rho} \leq 0$, $\frac{\partial\phi_e}{\partial\eta} \geq 0$, and $\frac{\partial\phi_e}{\partial Q} \geq 0$.

where; Π_{rz} = expected excess of market price of r, over future extraction costs of resource r in field z.

ρ_{rz} = expected excess of market price of r, over future import costs of resource r in field z.

η_{rz} = expected excess of market price of r, over capital costs.

Q_r = expected demand for resource r.

A set of equations paralleling (1)-(8) is thus required for "new fields" which result from exploration expenditures. The relations have the same interpretations as (1)-(8) but simply apply to field z=1,...,Z.

(11) $X^{t+1}_{rz} = X^t_{rz} - W^t_{rz} + h^t_{rz}$

(12) $W^t_{rz} = \phi_{wz}(\Pi^t_{rz}, \rho^t_{rz}, Q^t_r)$

(3') $Q^t_r = \phi_q(P_1, P_2, \ldots P_R)$

(13) $W^t_{rz} \leq g^t_{rz}(X^t_{rz}, K^t_z)$

(14) $S^{t+1}_{rz} = S^t_{rz} + W^t_{rz} + M^t_{rz} - \Delta^t_{rz}$

(15) $\Delta^t_{rz} = \phi_{\delta z}(\Pi^t_{rz=1,\ldots Z}, \rho^t_{rz=1,\ldots Z}, \mu^t_{rz=1,\ldots Z}, Q^t_r)$

(16) $M^t_{rz} = \phi_{mz}(\Pi^t_{rz=1,\ldots Z}, \rho^t_{rz=1,\ldots Z}, \mu^t_{rz=1,\ldots Z}, Q^t_r)$

(17) $S^t_{rz} \leq N^t_{rz}(\overline{K}^t_{rz})$

for all z, r and t.

Note that in (11), an initial stock X^0_{rz} is not given as in (1). If expenditures e^t_{rz} are made during the same period t, equation (11) becomes:

$$(11^1) \quad X^{t+1}_{rz} = h^t_{rz} - W^t_{rz} \text{ and in } t+1$$

$$(11^2) \quad X^{t+2}_{rz} = X^{t+1}_{rz} - W^{t+1}_{rz} + h^{t+1}_{rz}$$

and a "new field" z is brought into the model.

There are two general types of capital stocks which are used in the system. Those used for the extraction of resources in each field j and z, K^t_z, K^t_z, and those stocks used for the management and storage of stockpiles of each r in each field j and z, $\overline{K}^t_{rj}$, K^t_{rz}. In each field j and z, K^t_j and K^t_z are 1×N vectors, where K^t_{nj}, K^t_{nz}, n=1,2,...,N are intended to reflect, first a quantitative measure of various specific capital items (pumps, buildings, machinery, etc.) and second, a qualitative measure of capital items in terms of technologies. Thus, $K^t_{\bar{n}j}$ may be a petroleum refinery with hydrocracking capabilities, and K^t_{nj} a physical plant identical to $K^t_{\bar{n}j}$ except that hydrocracking capabilities do not exist. For any n, $\bar{n}$, marginal investments, substantively different from normal capital-formation notion of investment (to be denoted as V^t_j, V^t_z, later in this model), may have the effect of "converting" $K^t_{\bar{n}j}$ capital stocks to K^t_{nj} capital stocks, where such "conversions" have the effect of "converting" $K^t_{\bar{n}j}$ capital stocks to K^t_{nj} capital stocks, where such "conversion" have the effect of changing the technological structure of the extraction processing process.

Let $\sigma^t_{\bar{n}\ nj}$ be the investment expenditures which converts K_{nj} capital stocks to $K_{\bar{n}j}$ stocks, $\sigma^t_{n\ \bar{n}j}$ to convert $K_{\bar{n}j}$ to K_{nj}, $\sigma^t_{\bar{n}\ nz}$ the investment expenditures which converts K_{nz} capital stocks to $K_{\bar{n}z}$ stocks, and $\sigma^t_{n\ \bar{n}z}$ to convert $K_{\bar{n}z}$ to K_{nz} capital stocks. Assuming the conversion expenditures to have the functional form.

(18) $\sigma^t_{\bar{n}\ nj} = \phi_{\sigma\bar{j}}(\Pi^t_j,\ \eta^t_j,\ Q^t_r)$

(19) $\sigma^t_{n\ \bar{n}j} = \phi_{\sigma j}(\Pi^t_j,\ \eta^t_j,\ Q^t_r)$

(20) $\sigma^t_{\bar{n}\ nz} = \phi_{\sigma\bar{z}}(\Pi^t_z,\ \eta^t_z,\ Q^t_r)$

(21) $\sigma^t_{n\ \bar{n}z} = \phi_{\sigma z}(\Pi^t_z,\ \eta^t_z,\ Q^t_r)$

where; $\Pi^t_j = \sum_{r=1}^{R} (\Pi^t_{rj}),\quad \Pi^t_z = \sum_{r=1}^{R} (\Pi^t_{rz}),$

$$\eta^t_j = \sum_{r=1}^{R} (\eta^t_{rj}),\quad \eta^t_z = \sum_{r=1}^{R} (\eta^t_{rz}).$$

If $f=\phi_f(\sigma)$ is the quantitative measure of the resulting changes in the capital stocks after conversion expenditure σ, then

(22) $f^t_{\bar{n}\ nj} = \phi_{f\bar{j}}(\sigma^t_{\bar{n}\ nj})$

(23) $f^t_{n\ \bar{n}j} = \phi_{fj}(\sigma^t_{n\ \bar{n}j})$

(24) $f^t_{\bar{n}\ nz} = \phi_{f\bar{z}}(\sigma^t_{\bar{n}\ nz})$

(25) $f^t_{n\ \bar{n}z} = \phi_{fz}(\sigma^t_{n\ \bar{n}z})$

Define V^t as additional investment to capital stocks which are used for the extraction of resources in each field, and $\bar{V}^t$ as additional investments to capital stocks which are used for the management, import and storage of stockpiles,

and having the general formats:

$$(26) \quad V^t_{nj} = \phi_{vj}(\Pi^t_j, \rho^t_j, \mu^t_j, \eta^t_j, Q^t_r)$$

$$(27) \quad V^t_{nz} = \phi_{vz}(\Pi^t_z, \rho^t_z, \mu^t_z, \eta^t_z, Q^t_r)$$

$$(28) \quad \overline{V}^t_{rj} = \phi_{\overline{v}j}(\Pi^t_{rj}, \rho^t_{rj}, \mu^t_{rj}, \eta^t_{rj}, Q^t_r)$$

$$(29) \quad \overline{V}^t_{rz} = \phi_{\overline{v}z}(\Pi^t_{rz}, \rho^t_{rz}, \mu^t_{rz}, \eta^t_{rz}, Q^t_r)$$

Where; $\rho^t_j = \sum_{r=1}^{R} (\rho^t_{rj})$,

$$\mu^t_j = \sum_{r=1}^{R} (\mu^t_{rj}),$$

$$\rho^t_z = \sum_{r=1}^{R} (\rho^t_{rz}),$$

and $\mu^t_z = \sum_{r=1}^{R} (\mu^t_{rz})$.

Also define in general format:

$$(30) \quad D^t_{nj} = \phi_{dj}(W^t_{rj}, V^t_{nj}, K^t_{nj})$$

$$(31) \quad D^t_{nz} = \phi_{dz}(W^t_{rz}, V^t_{nz}, K^t_{nz})$$

$$(32) \quad \overline{D}^t_{rj} = \phi_{d\overline{j}}(\overline{V}^t_{rj}, \overline{K}^t_{rj})$$

$$(33) \quad \overline{D}^t_{rz} = \phi_{d\overline{z}}(\overline{V}^t_{rz}, \overline{K}^t_{rz})$$

where D as a net depreciation function for extraction capital stocks, with $\frac{\partial D}{\partial W} \geq 0$, $\frac{\partial D}{\partial V} \leq 0$, $\frac{\partial D}{\partial K} \geq 0$, and $\overline{D}$ as a net depreciation function of capital stocks used for management of stockpiles with $\frac{\partial \overline{D}}{\partial \overline{V}} \leq 0$, $\frac{\partial \overline{D}}{\partial \overline{K}} \geq 0$. Then the following transition equations are assumed to describe the time path for capital stocks:

(34) $K_{nj}^{t+1} = K_{nj}^{t} - D_{nj}^{t} + \sum_{\substack{\bar{n}=1 \\ \bar{n}\neq n}}^{N} (f_{n\,\bar{n}j}^{t} - f_{\bar{n}\,nj}^{t}),$

K_{nj}^{0} given, for all n, $\bar{n}$=1,...,N; j=1,...,J; t=1,...T.

(35) $K_{nz}^{t+1} = K_{nz}^{t} - D_{nz}^{t} + \sum_{\substack{\bar{n}=1 \\ \bar{n}\neq n}}^{N} (f_{n\,\bar{n}z}^{t} - f_{\bar{n}\,nz}^{t}),$

for all n, $\bar{n}$=1,...,N; j=1,...,J; t=1,...,T.

(36) $\bar{K}_{rj}^{t+1} = \bar{K}_{rj}^{t} - \bar{D}_{rj}^{t}$, for all j=1,...,J; t=1,...,T,

$\bar{K}_{rj}^{0}$ given

(37) $\bar{K}_{rz}^{t+1} = \bar{K}_{rz}^{t} - \bar{D}_{rz}^{t}$, for all z=1,...,Z; t=1,...,T.

A word of explanation may be required at this point in order to clarify the meaning of (34)-(37) above. The first two of the transition equations specify that the capital stock of type n available for use in extraction field j (or z) at any point in time depends on the size of the stock in preceding period less net depreciation over the preceding period (which may actually be negative if investment in new stocks is large and/or if extractions are small), plus the net conversion of other types of capital stocks to type n. Thus, the latter summation term takes account of the amounts of other classes of extractive capital ($\bar{n}\neq n$, $\bar{n}$=1,...,N) converted to type n, less that of type n, converted to other types.

Equations (36) and (37) are required for the available capital used to manage and store the, up to R, different energy resources in field j (or z) at time t+1. Since it is

essentially assumed homogeneity of this class of capital for any given type of energy resource, there are no conversions to quantify.

Next define U_r^t as the quantity of energy resource r available for use in the production of "final energy commodities" at time t.

Thus U_r^t may be thought of as the volume of petroleum (crude) made available to refineries, coal at trackside, etc.

The term U_r^t then includes the sum of draw downs from stockpiles of r from all resource fields, j, and z, for current consumption, then;

$$(38) \quad U_r^t = \sum_{j=1}^{J} \Delta_{rj}^t + \sum_{z=1}^{Z} \Delta_{rz}^t, \text{ for all } r=1,\ldots,R;\ t=1,\ldots,T.$$

Next it is assumed, there exist L different "final energy commodities" and define A_l^t as the amount of the l^{th} type produced at time t, $(l=1,\ldots,L)$. The amount of each produced depends upon the amount of the R possible energy resources used as inputs "U_{rl}^t", as well as up to Q different kinds of capital stocks "$\hat{K}_{ql}^t$" used in the production process. Thus the production functions could be written as:

$$(39) \quad A_l^t = \phi_l(U_{1l}^t,\ldots,U_{Rl}^t;\ \hat{K}_{1l}^t,\ldots,\hat{K}_{Ql}^t), \text{ for all } l=1,\ldots,L;\ t=1,\ldots,T.$$

with $\dfrac{\partial A_l^t}{\partial U_l^t} \geq 0$; $\dfrac{\partial A_l^t}{\partial \hat{K}_l^t} \geq 0$ for all r, q.

Note that obviously all energy resources might not be used as inputs in each production function process; then $\frac{\partial A}{\partial U_{rl}}$ may be zero for some r in each l.

Now let each unit of energy commodity 1 contain α_1 BTU's. If E^t equals the minimum amount of BTU required in period t, then the problem below must be constrained by:

$$(40) \quad \sum_{l=1}^{L} {}_{l}A_l^t \geq E^t, \text{ for all } t=1,\ldots,T.$$

Let $H_l^t \geq 0$ equal the minimum required quantity of the l^{th} specific energy commodity in period t. This implies.

$$(41) \quad A_l^t \geq H_l^t, \text{ for all } l=1,\ldots,L;\ t=1,\ldots,T.$$

C. THE CRITERION FUNCTION

The energy independence policy presented by President Nixon has been used as an application of the model introduced in previous section and for the formulation of its criterion function.

The basic concept of this policy arises from the threats to the security of the nation because of foreign supply interruptions, and the assumption that the threats most likely to be encountered, are best met through domestic self-sufficiency. Therefore, it calls for more storage of resources, their higher production, and expansion of explorations for new deposits and sources of energy, aimed toward the realization of this concept by period T. (1980) Within the decision situation described by the conditions and restrictions (1)-(41) of preceding section, concern here is with the indirect selection of values for extraction rates of natural resources, the expenditure rate for exploration, rate of investment of capital stocks, rate of production of energy commodities to

respond to energy requirements and production such that the following expression, the present value of total capital outlays and consumer costs over "T" periods of time be minimized.

$$(42)\quad \text{Min} \sum_{t=1}^{T} C^t\beta^t + \psi_T(X^T, K^T)\beta^t$$

Subject to (1)-(41)

where; C^t = Sum of all capital outlays and consumer costs.

ψ_T = A concave terminal value function for all the natural resources and capital stocks. [23]

β = $1/(1+r)$, where r is the appropriate positive discount rate. Reflects the discounting factor.

The objective function in a detailed form could be written as:

$$(43)\quad \text{Min} \sum_{t=1}^{T} \{\sum_{rz} e^t_{rz}\beta^t + \sum_{nj} V^t_{nj}\beta^t + \sum_{nz} V^t_{nz}\beta^t + \sum_{rj} \bar{V}^t_{rj}\beta^t + \sum_{rz} \bar{V}^t_{rz}\beta^t$$

$$+\sum_{\bar{n}j} \sigma^t_{n\,\bar{n}j}\beta^t + \sum_{\bar{n}z} \sigma^t_{n\,\bar{n}z}\beta^t + \sum_{nj} \sigma^t_{\bar{n}\,nj}\beta^t + \sum_{nz} \sigma^t_{\bar{n}\,nz}\beta^t$$

$$+\sum_{rj} W^t_{rj}\cdot C^{wt}_{rj}\beta^t + \sum_{rz} W^t_{rz}\cdot C^{wt}_{rz}\beta^t + \sum_{rj} S^t_{rj}\cdot C^{st}_{rj}\beta^t$$

$$+\sum_{rz} S^t_{rz}\cdot C^{st}_{rz}\beta^t + \sum_{rj} M^t_{rj}\cdot C^{mt}_{rj}\beta^t + \sum_{rz} M^t_{rz}\cdot C^{mt}_{rz}\beta^t\}$$

$$+\psi_T(X^T, K^T)\beta^T.$$

Subject to equations (1)-(41)

where; $\sum_{rz} = \sum_{r=1}^{R}\sum_{z=1}^{Z}$; $\sum_{rj} = \sum_{r=1}^{R}\sum_{j=1}^{J}$; $\sum_{\bar{n}j} = \sum_{\bar{n}=1}^{N}\sum_{j=1}^{J}$

$X^T = \sum_{rj} X^T_{rj} + \sum_{rz} X^T_{rz}$

$K^T = \sum_j K^T_j + \sum_z K^T_z + \sum_{rj} \bar{K}^T_{rj} + \sum_{rz} \bar{K}^T_{rz} + \sum_{1q} \hat{K}^T_{1q}.$

D. EQUILIBRIUM CONDITIONS

The optimization problem expressed by (43) and equations (1) through (41) could be solved by Lagrangian method. Assuming that the functions in (1) through (43) are continuous and possess first partial derivatives with respect to each argument, the following Lagrangian expression may be formed.

$$
\begin{aligned}
(44)\quad L = \sum_{t=1}^{T} \{ & \sum_{rz} e^t_{rz}\beta^t + \sum_{nj} V^t_{nj}\beta^t + \sum_{nz} V^t_{nz}\beta^t + \sum_{rj} \bar{V}^t_{rj}\beta^t + \sum_{rz} \bar{V}^t_{rz}\beta^t \\
& + \sum_{\bar{n}j} \sigma^t_{n\,\bar{n}j}\beta^t + \sum_{\bar{n}j} \sigma^t_{n\,\bar{n}z}\beta^t + \sum_{nj} \sigma^t_{\bar{n}\,nj}\beta^t + \sum_{nz} \sigma^t_{\bar{n}\,nz}\beta^t \\
& + \sum_{rj} W^t_{rj}\cdot C^{wt}_{rj}\beta^t + \sum_{rz} W^t_{rz}\cdot C^{wt}_{rz}\beta^t + \sum_{rj} S^t_{rj}\cdot C^{st}_{rj}\beta^t + \sum_{rz} S^t_{rz}\cdot C^{st}_{rz}\beta^t \\
& + \sum_{rj} M^t_{rj}\cdot C^{mt}_{rj}\beta^t + \sum_{rz} M^t_{rz}\cdot C^{mt}_{rz}\beta^t - {}^{1}\lambda^t_{rj}[X^t_{rj} - X^0_{rj} + \sum_{\tau=1}^{t} W^t_{rj}] \\
& - \sum_{rj} {}^{2}\lambda^t_{rj}[W^t_{rj} - \phi^t_{wj}(\Pi^t_{rj},\ \rho^t_{rj},\ Q^t_r)] \\
& - \sum_{r} {}^{3}\lambda^t_{r}[Q^t_r - \phi^t_q(P_1,\ P_2, \ldots .P_Q)] \\
& - \sum_{rj} {}^{4}\lambda^t_{rj}[W^t_{rj} - g^t_{rj}(X^t_{rj},\ K^t_j)] \\
& - {}^{5}\lambda^t_{rj}[S^t_{rj} - S^0_{rj} - \sum_{\tau=1}^{t} W^t_{rj} - \sum_{\tau=1}^{t} M^t_{rj} + \sum_{\tau=1}^{t} \Delta^t_{rj}] \\
& - \sum_{rj} {}^{6}\lambda^t_{rj}[\Delta^t_{rj} - \phi^t_{\delta j}(\Pi^t_{rj=1,\ldots J}, \rho^t_{rj=1,\ldots J}, \mu^t_{rj=1,\ldots J}, Q^t_r)] \\
& - \sum_{rj} {}^{7}\lambda^t_{rj}[M^t_{rj} - \phi^t_{mj}(\Pi^t_{rj=1,\ldots J}, \rho^t_{rj=1,\ldots J}, \mu^t_{rj=1,\ldots J}, Q^t_r)] \\
& - \sum_{rj} {}^{8}\lambda^t_{rj}[S^t_{rj} - N^t_{rj}(\bar{K}^t_{rj})] - {}^{9}\lambda^t_{rz}[h^t_{rz}(e^t_{rz}) - \bar{X}_{rz}] \\
& - \sum_{rz} {}^{10}\lambda^t_{rz}[e^t_{rz} - \phi_e(\Pi^t_{rz}, \rho^t_{rz}, \eta^t_{rz}, Q^t_r)]\cdot\beta^t \\
& - {}^{11}\lambda^t_{rz}[X^t_{rz} + \sum_{\tau=1}^{t} W^t_{rz} - \sum_{\tau=1}^{t} h^t_{rz}] \\
& - \sum_{rz} {}^{12}\lambda^t_{rz}[W^t_{rz} - \phi_{wz}(\Pi^t_{rz}, \rho^t_{rz}, Q^t_r)] \\
& - \sum_{rz} {}^{13}\lambda^t_{rz}[W^t_{rz} - g^t_{rz}(X^t_{rz},\ K^t_z)]
\end{aligned}
$$

$$-{}^{14}\lambda^t_{rz}[S^t_{rz}-\sum_{\tau=1}^{t} W^t_{rz}-\sum_{\tau=1}^{t} M^t_{rz}+\sum_{\tau=1}^{t} \Delta^t_{rz}]$$

$$-\sum_{rz} {}^{15}\lambda^t_{rz}[\Delta^t_{rz}-\phi^t_{\delta z}(\Pi^t_{rz=1,..Z},\rho^t_{rz=1,..Z},\mu^t_{rz=1,..Z},Q^t_r)]$$

$$-\sum_{rz} {}^{16}\lambda^t_{rz}[M^t_{rz}-\phi^t_{mz}(\Pi^t_{rz=1,..Z},\rho^t_{rz=1,..Z},\mu^t_{rz=1,..Z},Q^t_r)]$$

$$-\sum_{rz} {}^{17}\lambda^t_{rz}[S^t_{rz}-N^t_{rz}(\overline{K}^t_{rz})]-\sum_{nj} {}^{18}\lambda^t_{nj}\beta^t[\sigma^t_{\bar{n}\ nj}-\phi_{\sigma\bar{j}}(\Pi^t_j,\eta^t_j,Q^t_r)]$$

$$-\sum_{\bar{n}j} {}^{19}\lambda^t_{\bar{n}j}[\sigma^t_{n\ \bar{n}j}-\phi_{\sigma j}(\Pi^t_j,\eta^t_j,Q^t_r)]\beta^t$$

$$-\sum_{nz} {}^{20}\lambda^t_{nz}[\sigma^t_{\bar{n}\ nz}-\phi_{\sigma\bar{z}}(\Pi^t_z,\eta^t_z,Q^t_r)]\beta^t$$

$$-\sum_{\bar{n}z} {}^{21}\lambda^t_{\bar{n}z}[\sigma^t_{n\ \bar{n}z}-\phi_{\sigma z}(\Pi^t_z,\eta^t_z,Q^t_r)]\beta^t$$

$$-\sum_{nj} {}^{22}\lambda^t_{nj}[f^t_{\bar{n}\ nj}-\phi_{f\bar{j}}(\sigma^t_{\bar{n}\ nj})]-\sum_{\bar{n}j} {}^{23}\lambda^t_{\bar{n}j}[f^t_{n\ \bar{n}j}-\phi_{fj}(\sigma^t_{n\ \bar{n}j})]$$

$$-\sum_{nz} {}^{24}\lambda^t_{nz}[f^t_{\bar{n}\ nz}-\phi_{f\bar{z}}(\sigma^t_{\bar{n}\ nz})]-\sum_{\bar{n}z} {}^{25}\lambda^t_{\bar{n}z}[f^t_{n\ \bar{n}z}-\phi_{fz}(\sigma^t_{n\ \bar{n}z})]$$

$$-\sum_{nj} {}^{26}\lambda^t_{nj}\beta^t[V^t_{nj}-\phi_{vj}(\Pi^t_j,\rho^t_j,\mu^t_j,\eta^t_j,Q^t_r)]$$

$$-\sum_{nz} {}^{27}\lambda^t_{nz}\beta^t[V^t_{nz}-\phi_{vz}(\Pi^t_z,\rho^t_z,\mu^t_z,\eta^t_z,Q^t_z)]$$

$$-\sum_{rj} {}^{28}\lambda^t_{rj}\beta^t[\overline{V}^t_{rj}-\phi_{\bar{v}j}(\Pi^t_{rj},\rho^t_{rj},\mu^t_{rj},\eta^t_{rj},Q^t_r)]$$

$$-\sum_{rz} {}^{29}\lambda^t_{rz}\beta^t[\overline{V}^t_{rz}-\phi_{\bar{v}z}(\Pi^t_{rz},\rho^t_{rz},\mu^t_{rz},\eta^t_{rz},Q^t_r)]$$

$$-\sum_{nj} {}^{30}\lambda^t_{nj}[D^t_{nj}-\phi_{dj}(W^t_{rj},\ V^t_{nj},\ K^t_{nj})]$$

$$-\sum_{nz} {}^{31}\lambda^t_{nz}[D^t_{nz}-\phi_{dz}(W^t_{rz},\ V^t_{nz},\ K^t_{nz})]$$

$$-\sum_{rj} {}^{32}\lambda^t_{rj}[\overline{D}^t_{rj}-\phi_{d\bar{j}}(\overline{V}^t_{rj},\overline{K}^t_{rj})]$$

$$-\sum_{rz} {}^{33}\lambda^t_{rz}[\overline{D}^t_{rz}-\phi_{d\bar{z}}(\overline{V}^t_{rz},\overline{K}^t_{rz})]$$

$$-{}^{34}\lambda^t_{nj}[K^t_{nj}-K^0_{nj}-\sum_{\tau=1}^{t} D^t_{nj}-\sum_{\tau=1}^{t}\ \sum_{\bar{n}=1}^{N}(f^t_{n\ \bar{n}j}-f^t_{\bar{n}\ nj})]$$

$$-{}^{35}\lambda^t_{nz}[K^t_{nz}-\sum_{\tau=1}^{t} D^t_{nz}-\sum_{\tau=1}^{t}\ \sum_{\bar{n}=1}^{N}(f^t_{n\ \bar{n}z}-f^t_{\bar{n}\ nz})]$$

$$-{}^{36}\lambda^t_{rj}[\bar{K}^t_{rj}-\bar{K}^0_{rj}+\sum_{\tau=1}^{t}\bar{D}^t_{rj}]-{}^{37}\lambda^t_{rz}[\bar{K}^t_{rz}+\sum_{\tau=1}^{t}\bar{D}^t_{rz}]\}$$

$$+\sum_{t=\bar{T}}^{T}\{-{}^{38}\lambda^t_1[A^t_1-\phi_1(U^t_{r1},\ldots U^t_{R1};\ \hat{K}^t_{11},\ldots\hat{K}^t_{Q1})]$$

$$-{}^{39}\lambda^t_1[\sum_{1=1}^{L}\alpha_1A^t_1-E^t]-{}^{40}\lambda^t_1[A^t_1-H^t_1]\}+\ \psi_T(X^T,K^T)\beta^t.$$

The necessary conditions for minimizing equation (44) include the following equations, which are then respectively interpreted for their economic implications on the objective function.

(45) $$\frac{\partial L}{\partial V^t_{nj}} = \beta^t-{}^{26}\lambda^t_{nj}\beta^t-{}^{30}\lambda^t_{nj}[\frac{\partial\phi^t_{dj}}{\partial V^t_{nj}}]$$

In equation (45) the term ${}^{26}\lambda^t_{nj}\beta^t$ represents the discounted value of the incremental change in objective function due to a unit of change in the investment of capital stocks used for extraction of resource r in field j, defined by constraining equation (26). The terms ${}^{30}\lambda^t_{nj}[\frac{\partial\phi^t_{dj}}{\partial V^t_{nj}}]$ represents the implicit value of a marginal change in net depreciation of these capital stocks due to a marginal change in relative investments. The necessary condition for optimality requires that $\frac{\partial L}{\partial V^t_{nj}} = 0$, or

(45-1) $${}^{30}\lambda^t_{nj}[\frac{\partial\phi^t_{dj}}{\partial V^t_{nj}}] = \beta^t-{}^{26}\lambda^t_{nj}\beta^t.$$

A similar interpretation can be made for equations (46)-(52) below.

$$(46)\quad \frac{\partial L}{\partial V^t_{nz}} = \beta^t - {}^{27}\lambda^t_{nz}\beta^t - {}^{31}\lambda^t_{nz}\left[\frac{\partial \phi^t_{dz}}{\partial V^t_{nz}}\right] = 0$$

$$(46\text{-}1)\quad {}^{31}\lambda^t_{nz}\left[\frac{\partial \phi^t_{dz}}{\partial V^t_{nz}}\right] = \beta^t - {}^{27}\lambda^t_{nz}\beta^t$$

$$(47)\quad \frac{\partial L}{\partial \overline{V}^t_{rj}} = \beta^t - {}^{28}\lambda^t_{rj}\beta^t - {}^{32}\lambda^t_{rj}\left[\frac{\partial \phi_{d\bar{j}}}{\partial \overline{V}^t_{rj}}\right] = 0$$

$$(47\text{-}1)\quad {}^{32}\lambda^t_{rj}\left[\frac{\partial \phi_{d\bar{j}}}{\partial \overline{V}^t_{rj}}\right] = \beta^t - {}^{28}\lambda^t_{rj}\beta^t$$

$$(48)\quad \frac{\partial L}{\partial \overline{V}^t_{rz}} = \beta^t - {}^{29}\lambda^t_{rz}\beta^t - {}^{33}\lambda^t_{rz}\left[\frac{\partial \phi^t_{d\bar{z}}}{\partial \overline{V}^t_{rz}}\right] = 0$$

$$(48\text{-}1)\quad {}^{33}\lambda^t_{rz}\left[\frac{\partial \phi^t_{d\bar{z}}}{\partial \overline{V}^t_{rz}}\right] = \beta^t - {}^{29}\lambda^t_{rz}\beta^t$$

$$(49)\quad \frac{\partial L}{\partial \sigma^t_{n\,\bar{n}j}} = \beta^t - {}^{19}\lambda^t_{nj}\beta^t + {}^{23}\lambda^t_{\bar{n}j}\left[\frac{\partial \phi^t_{fj}}{\partial \sigma^t_{n\,\bar{n}j}}\right] = 0$$

$$(49\text{-}1)\quad {}^{23}\lambda^t_{\bar{n}j}\left[\frac{\partial \phi^t_{fj}}{\partial \sigma^t_{n\,\bar{n}j}}\right] = {}^{19}\lambda^t_{nj}\beta^t - \beta^t$$

$$(50)\quad \frac{\partial L}{\partial \sigma^t_{n\,\bar{n}z}} = 0$$

$$(50\text{-}1)\quad {}^{25}\lambda^t_{\bar{n}z}\left[\frac{\partial \phi^t_{fz}}{\partial \sigma^t_{n\,\bar{n}z}}\right] = \beta^t\left[{}^{21}\lambda^t_{\bar{n}z} - 1\right]$$

$$(51)\quad \frac{\partial L}{\partial \sigma^t_{\bar{n}\,nj}} = 0$$

$$(51\text{-}1)\quad {}^{22}\lambda^t_{nj}\left[\frac{\partial \phi^t_{f\bar{j}}}{\partial \sigma^t_{\bar{n}\,nj}}\right] = \beta^t\left[{}^{18}\lambda^t_{nj} - 1\right]$$

(52) $\frac{\partial L}{\partial \sigma^{t}_{\bar{n}\ nz}} = 0$

(52-1) $^{24}\lambda^{t}_{nz}[\frac{\partial \phi^{t}_{f\bar{z}}}{\partial \sigma^{t}_{\bar{n}\ nz}}] = \beta^{t}[^{20}\lambda^{t}_{nz}-1]$

(53) $\frac{\partial L}{\partial W^{t}_{rj}} = \beta^{t}\cdot C^{wt}_{rj}-{}^{1}\lambda^{t}_{rj}-{}^{2}\lambda^{t}_{rj}-{}^{4}\lambda^{t}_{rj}+{}^{5}\lambda^{t}_{rj}-{}^{30}\lambda^{t}_{nj}[\frac{\partial \phi^{t}_{dj}}{\partial W^{t}_{rj}}] = 0$

In equation (53) the term $\beta^{t}\cdot C^{wt}_{rj}$ represents the discounted value of the extraction cost, and $^{30}\lambda^{t}_{nj}[\frac{\partial \phi^{t}_{dj}}{\partial W^{t}_{rj}}]$ represents the implicit value of the marginal change in net depreciation of the capital stocks due to a marginal change in extraction rate of resource r, where $^{1}\lambda^{t}_{rj}$ represents the value of the incremental change in objective function due to a unit of change in the extraction rate restricted by equation (1) and $^{2}\lambda^{t}_{rj}$ for equation (2) and $^{4}\lambda^{t}_{rj}$ for equation (4) and $^{5}\lambda^{t}_{rj}$ for equation (5). The necessary condition for optimality requires that

(53-1) $^{30}\lambda^{t}_{nj}[\frac{\partial \phi^{t}_{dj}}{\partial W^{t}_{rj}}] = \beta^{t}\cdot C^{wt}_{rj}-({}^{1}\lambda^{t}_{rj}+{}^{2}\lambda^{t}_{rj}+{}^{4}\lambda^{t}_{rj}-{}^{5}\lambda^{t}_{rj})$

A similar interpretation could be made for the equation (54) below.

(54) $\frac{\partial L}{\partial W^{t}_{rz}} = \beta^{t}\cdot C^{wt}_{rz}-{}^{11}\lambda^{t}_{rz}-{}^{12}\lambda^{t}_{rz}-{}^{13}\lambda^{t}_{rz}+{}^{14}\lambda^{t}_{rz}-{}^{31}\lambda^{t}_{rz}[\frac{\partial \phi_{dz}}{\partial W^{t}_{rz}}] = 0$

(54-1) $^{31}\lambda^{t}_{rz}[\frac{\partial \phi_{dz}}{\partial W^{t}_{rz}}] = \beta^{t}\cdot C^{wt}_{rz}-({}^{11}\lambda^{t}_{rz}+{}^{12}\lambda^{t}_{rz}+{}^{13}\lambda^{t}_{rz}-{}^{14}\lambda^{t}_{rz})$

(55) $\frac{\partial L}{\partial e^{t}_{rz}} = \beta^{t}-{}^{9}\lambda^{t}_{rz}[\frac{\partial h^{t}_{rz}}{\partial e^{t}_{rz}}]-{}^{10}\lambda^{t}_{rz}\beta^{t}+{}^{11}\lambda^{t}_{rz} = 0$

In equation (55) the term ${}^{9}\lambda^t_{rz}[\frac{\partial h^t_{rz}}{\partial e^t_{rz}}]$ represents the implicit value of a marginal change in units discovered of the resource r in field z due to a marginal change in the exploration expenditure restricted by equation (9). The term ${}^{10}\lambda^t_{rz}\beta^t$ represents the discounted value of the incremental change in objective function due to a unit change in exploration expenditure described by equation (10). Similarly the term ${}^{11}\lambda^t_{rz}$ represents the value of an incremental change in objective function due to a unit change in the quantity discovered of resource r described by equation (11). The necessary condition for optimality requires that

$$(55\text{-}1) \quad {}^{9}\lambda^t_{rz}[\frac{\partial h^t_{rz}}{\partial e^t_{rz}}] = \beta^t(1-{}^{10}\lambda^t_{rz})+{}^{11}\lambda^t_{rz}$$

The most important consideration in this study, is the impact of price changes, on the objective function and other variables in the model. This is described by (56), which is the partial derivative of equation (44) with respect to P^t_r, the price of resource r in period t,

$$(56) \quad \frac{\partial L}{\partial P^t_r} = {}^{2}\lambda^t_{rj}[\frac{\partial \phi^t_{wj}}{\partial \Pi^t_{rj}}.\frac{\partial \Pi^t_{rj}}{\partial P^t_r} + \frac{\partial \phi^t_{wj}}{\partial \rho^t_{rj}}.\frac{\partial \rho^t_{rj}}{\partial P^t_r}]+{}^{3}\lambda^t_{r}[\frac{\partial \phi^t_{q}}{\partial P^t_r}]$$

$$+{}^{6}\lambda^t_{rj}[\frac{\partial \phi^t_{\delta j}}{\partial \Pi^t_{rj}}.\frac{\partial \Pi^t_{rj}}{\partial P^t_r} + \frac{\partial \phi^t_{\delta j}}{\partial \rho^t_{rj}}.\frac{\partial \rho^t_{rj}}{\partial P^t_r} + \frac{\partial \phi^t_{\delta j}}{\partial \mu^t_{rj}}.\frac{\partial \mu^t_{rj}}{\partial P^t_r}]$$

$$+{}^{7}\lambda^t_{rj}[\frac{\partial \phi^t_{mj}}{\partial \Pi^t_{rj}}.\frac{\partial \Pi^t_{rj}}{\partial P^t_r} + \frac{\partial \phi^t_{mj}}{\partial \rho^t_{rj}}.\frac{\partial \rho^t_{rj}}{\partial P^t_r} + \frac{\partial \phi^t_{mj}}{\partial \mu^t_{rj}}.\frac{\partial \mu^t_{rj}}{\partial P^t_r}]$$

$$+{}^{10}\lambda_{rz}^{t}\left[\frac{\partial\phi_e^t}{\partial\Pi_{rz}^t}\cdot\frac{\partial\Pi_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_e^t}{\partial\rho_{rz}^t}\cdot\frac{\partial\rho_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_e^t}{\partial\eta_{rz}^t}\cdot\frac{\partial\eta_{rz}^t}{\partial P_r^t}\right]\beta^t$$

$$+{}^{12}\lambda_{rz}^{t}\left[\frac{\partial\phi_{wz}^t}{\partial\Pi_{rz}^t}\cdot\frac{\partial\Pi_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{wz}^t}{\partial\rho_{rz}^t}\cdot\frac{\partial\rho_{rz}^t}{\partial P_r^t}\right]$$

$$+{}^{15}\lambda_{rz}^{t}\left[\frac{\partial\phi_{\delta z}^t}{\partial\Pi_{rz}^t}\cdot\frac{\partial\Pi_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{\delta z}^t}{\partial\rho_{rz}^t}\cdot\frac{\partial\rho_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{\delta z}^t}{\partial\mu_{rz}^t}\cdot\frac{\partial\mu_{rz}^t}{\partial P_r^t}\right]$$

$$+{}^{16}\lambda_{rz}^{t}\left[\frac{\partial\phi_{mz}^t}{\partial\Pi_{rz}^t}\cdot\frac{\partial\Pi_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{mz}^t}{\partial\rho_{rz}^t}\cdot\frac{\partial\rho_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{mz}^t}{\partial\mu_{rz}^t}\cdot\frac{\partial\mu_{rz}^t}{\partial P_r^t}\right]$$

$$+{}^{18}\lambda_{nj}^{t}\beta^t\left[\frac{\partial\phi_{\delta\bar{j}}}{\partial\Pi_{rj}^t}\cdot\frac{\partial\Pi_{rj}^t}{\partial P_r^t}+\frac{\partial\phi_{\delta\bar{j}}}{\partial\eta_{rj}^t}\cdot\frac{\partial\eta_{rj}^t}{\partial P_r^t}\right]$$

$$+{}^{19}\lambda_{\bar{n}j}^{t}\beta^t\left[\frac{\partial\phi_{\delta j}}{\partial\Pi_{rj}^t}\cdot\frac{\partial\Pi_{rj}^t}{\partial P_r^t}+\frac{\partial\phi_{\delta j}}{\partial\eta_{rj}^t}\cdot\frac{\partial\eta_{rj}^t}{\partial P_r^t}\right]$$

$$+{}^{20}\lambda_{nz}^{t}\beta^t\left[\frac{\partial\phi_{\delta\bar{z}}}{\partial\Pi_{rz}^t}\cdot\frac{\partial\Pi_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{\delta\bar{z}}}{\partial\eta_{rz}^t}\cdot\frac{\partial\eta_{rz}^t}{\partial P_r^t}\right]$$

$$+{}^{21}\lambda_{\bar{n}z}^{t}\beta^t\left[\frac{\partial\phi_{\delta z}}{\partial\Pi_{rz}^t}\cdot\frac{\partial\Pi_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{\delta z}}{\partial\eta_{rz}^t}\cdot\frac{\partial\eta_{rz}^t}{\partial P_r^t}\right]$$

$$+{}^{26}\lambda_{nj}^{t}\beta^t\left[\frac{\partial\phi_{vj}}{\partial\Pi_{rj}^t}\cdot\frac{\partial\Pi_{rj}^t}{\partial P_r^t}+\frac{\partial\phi_{vj}}{\partial\rho_{rj}^t}\cdot\frac{\partial\rho_{rj}^t}{\partial P_r^t}+\frac{\partial\phi_{vj}}{\partial\mu_{rj}^t}\cdot\frac{\partial\mu_{rj}^t}{\partial P_r^t}\right.$$

$$\left.+\frac{\partial\phi_{vj}}{\partial\eta_{rj}^t}\cdot\frac{\partial\eta_{r}^t}{\partial P_r^t}\right]+{}^{27}\lambda_{nz}^{t}\beta^t\left[\frac{\partial\phi_{vz}}{\partial\Pi_{rz}^t}\cdot\frac{\partial\Pi_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{vz}}{\partial\rho_{rz}^t}\cdot\frac{\partial\rho_{rz}^t}{\partial P_r^t}\right.$$

$$\left.+\frac{\partial\phi_{vz}}{\partial\mu_{rz}^t}\cdot\frac{\partial\mu_{rz}^t}{\partial P_r^t}+\frac{\partial\phi_{vz}}{\partial\eta_{rz}^t}\cdot\frac{\partial\eta_{rz}^t}{\partial P_r^t}\right]+{}^{28}\lambda_{rj}^{t}\beta^t\left[\frac{\partial\phi_{\bar{v}j}}{\partial\Pi_{rj}^t}\cdot\frac{\partial\Pi_{rj}^t}{\partial P_r^t}\right.$$

$$\left.+\frac{\partial\phi_{\bar{v}j}}{\partial\rho_{rj}^t}\cdot\frac{\partial\rho_{rj}^t}{\partial P_r^t}+\frac{\partial\phi_{\bar{v}j}}{\partial\mu_{rj}^t}\cdot\frac{\partial\mu_{rj}^t}{\partial P_r^t}+\frac{\partial\phi_{\bar{v}j}}{\partial\eta_{rj}^t}\cdot\frac{\partial\eta_{rj}^t}{\partial P_r^t}\right]$$

$$+{}^{29}\lambda^t_{rz}\beta^t\left[\frac{\partial\phi_{\bar{v}z}}{\partial\Pi^t_{rz}}\cdot\frac{\partial\Pi^t_{rz}}{\partial P^t_r} + \frac{\partial\phi_{\bar{v}z}}{\partial\rho^t_{rz}}\cdot\frac{\partial\rho^t_{rz}}{\partial P^t_r} + \frac{\partial\phi_{\bar{v}z}}{\partial\mu^t_{rz}}\cdot\frac{\partial\mu^t_{rz}}{\partial P^t_r}\right.$$

$$\left. + \frac{\partial\phi_{\bar{v}z}}{\partial\eta^t_{rz}}\cdot\frac{\partial\eta^t_{rz}}{\partial P^t_r}\right] = 0$$

And after the arrangement of the terms, it could be written

$$(56\text{-}1)\quad \frac{\partial\Pi^t_{rj}}{\partial P^t_r}\left[{}^{2}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{wj}}{\partial\Pi^t_{rj}} + {}^{6}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{\delta j}}{\partial\Pi^t_{rj}} + {}^{7}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{nj}}{\partial\Pi^t_{rj}}\right.$$

$$\left.+{}^{18}\lambda^t_{nj}\beta^t\cdot\frac{\partial\phi_{\sigma\bar{j}}}{\partial\Pi^t_{rj}} + {}^{19}\lambda^t_{\bar{n}j}\beta^t\cdot\frac{\partial\phi_{\sigma j}}{\partial\Pi^t_{rj}}\right] + \frac{\partial\Pi^t_{rz}}{\partial P^t_r}\left[{}^{10}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{e}}{\partial\Pi^t_{rz}}\right.$$

$$+{}^{12}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{wz}}{\partial\Pi^t_{rz}} + {}^{15}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{\delta z}}{\partial\Pi^t_{rz}} + {}^{16}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{mz}}{\partial\Pi^t_{rz}} + {}^{20}\lambda^t_{nz}\cdot\frac{\partial\phi_{\sigma\bar{z}}}{\partial\Pi^t_{rz}}$$

$$\left.+{}^{21}\lambda^t_{\bar{n}z}\beta^t\cdot\frac{\partial\phi_{\sigma z}}{\partial\Pi^t_{rz}} + {}^{27}\lambda^t_{nz}\beta^t\cdot\frac{\partial\phi_{vz}}{\partial\Pi^t_{rz}} + {}^{29}\lambda^t_{rz}\beta^t\cdot\frac{\partial\phi_{\bar{v}z}}{\partial\Pi^t_{rz}}\right]$$

$$+ \frac{\partial\rho^t_{rj}}{\partial P^t_r}\left[{}^{2}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{wj}}{\partial\rho^t_{rj}} + {}^{6}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{\delta j}}{\partial\rho^t_{rj}} + {}^{7}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{mj}}{\partial\rho^t_{rj}}\right.$$

$$\left.+{}^{26}\lambda^t_{nj}\beta^t\cdot\frac{\partial\phi_{vj}}{\partial\rho^t_{rj}} + {}^{28}\lambda^t_{rj}\beta^t\cdot\frac{\partial\phi_{\bar{v}j}}{\partial\rho^t_{rj}}\right]$$

$$+ \frac{\partial\mu^t_{rj}}{\partial P^t_r}\left[{}^{6}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{\delta j}}{\partial\mu^t_{rj}} + {}^{7}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{mj}}{\partial\mu^t_{rj}} + {}^{26}\lambda^t_{nj}\beta^t\cdot\frac{\partial\phi_{vj}}{\partial\mu^t_{rj}}\right.$$

$$\left.+{}^{28}\lambda^t_{rj}\beta^t\cdot\frac{\partial\phi_{\bar{v}j}}{\partial\mu^t_{rj}}\right] + \frac{\partial\eta^t_{rj}}{\partial P^t_r}\left[{}^{10}\lambda^t_{rj}\cdot\frac{\partial\phi^t_{e}}{\partial\eta^t_{rz}} + {}^{18}\lambda^t_{nj}\beta^t\cdot\frac{\partial\phi_{\sigma\bar{j}}}{\partial\eta^t_{rj}}\right.$$

$$\left.+{}^{19}\lambda^t_{\bar{n}j}\beta^t\cdot\frac{\partial\phi_{\sigma j}}{\partial\eta^t_{rj}} + {}^{26}\lambda^t_{nj}\beta^t\cdot\frac{\partial\phi_{vj}}{\partial\eta^t_{rj}} + {}^{28}\lambda^t_{rj}\beta^t\cdot\frac{\partial\phi_{\bar{v}j}}{\partial\eta^t_{rj}}\right]$$

$$+\frac{\partial\rho^t_{rz}}{\partial P^t_r}[{}^{10}\lambda^t_{rz}\cdot\frac{\partial\phi^t_e}{\partial\rho^t_{rz}}+{}^{12}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{wz}}{\partial\rho^t_{rz}}+{}^{15}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{\delta z}}{\partial\rho^t_{rz}}$$

$$+{}^{16}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{mz}}{\partial\rho^t_{rz}}+{}^{27}\lambda^t_{nz}\beta^t\cdot\frac{\partial\phi_{vz}}{\partial\rho^t_{rz}}+{}^{29}\lambda^t_{rz}\beta^t\cdot\frac{\partial\phi_{\bar{v}z}}{\partial\rho^t_{rz}}]$$

$$+\frac{\partial\mu^t_{rz}}{\partial P^t_r}[{}^{15}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{\delta z}}{\partial\mu^t_{rz}}+{}^{16}\lambda^t_{rz}\cdot\frac{\partial\phi^t_{mz}}{\partial\mu^t_{rz}}+{}^{27}\lambda^t_{rz}\beta^t\cdot\frac{\partial\phi_{vz}}{\partial\mu^t_{rz}}$$

$$+{}^{29}\lambda^t_{rz}\beta^t\cdot\frac{\partial\phi_{\bar{v}z}}{\partial\mu^t_{rz}}]+\frac{\partial\eta^t_{rz}}{\partial P^t_r}[{}^{10}\lambda^t_{rz}\cdot\frac{\partial\phi^t_e}{\partial\eta^t_{rz}}+{}^{20}\lambda^t_{rz}\beta^t\cdot\frac{\partial\phi_{\sigma\bar{z}}}{\partial\eta^t_{rz}}$$

$$+{}^{21}\lambda^t_{\bar{n}z}\beta^t\cdot\frac{\partial\phi_{\sigma z}}{\partial\eta^t_{rz}}+{}^{27}\lambda^t_{nz}\beta^t\cdot\frac{\partial\phi_{vz}}{\partial\eta^t_{rz}}+{}^{29}\lambda^t_{rz}\beta^t\cdot\frac{\partial\phi_{\bar{v}z}}{\partial\eta^t_{rz}}]=$$

$$-{}^{3}\lambda^t_r[\frac{\partial\phi^t_q}{\partial P^t_r}].$$

In equation (56-1) the terms outside of the brackets represent the marginal change in the excess of markets price of resource r over different costs in different fields due to a marginal change in the market price of resource r. And the terms inside the brackets (some of them discounted) represent the implicit value of the marginal changes, in different variables, influenced by the price of resource r, due to a marginal change in the excess price over the costs.

In optimality the total sum of these terms in the left hand side of equation (56-1) must be equal to the term on the right hand side $-{}^{3}\lambda^t_r[\frac{\partial\phi^t_q}{\partial P^t_r}]$ which represents the implicit value of the marginal change in units demanded of resource

r, due to a marginal change in its market price, the demand function is described by equation (3).

The above mentioned conditions simultaneously solved with the following equations would determine the values of multipliers and the optimum rates of exploration, investment, extraction, importation and conversion expenditure.

(57) $\frac{\partial L}{\partial M^t_{rj}} = \beta^t \cdot C^{mt}_{rj} + {}^5\lambda^t_{rj} - {}^7\lambda^t_{rj} = 0$

(57-1) ${}^5\lambda^t_{rj} - {}^7\lambda^t_{rj} = -\beta^t \cdot C^{mt}_{rj}$

(58) $\frac{\partial L}{\partial M^t_{rz}} = \beta^t \cdot C^{mt}_{rz} + {}^{14}\lambda^t_{rz} - {}^{16}\lambda^t_{rz} = 0$

(58-1) ${}^{16}\lambda^t_{rz} - {}^{14}\lambda^t_{rz} = \beta^t \cdot C^{mt}_{rz}$

(59) $\frac{\partial L}{\partial S^t_{rj}} = \beta^t \cdot C^{st}_{rj} - {}^5\lambda^t_{rj} - {}^8\lambda^t_{rj} = 0$

(59-1) ${}^5\lambda^t_{rj} + {}^8\lambda^t_{rj} = \beta^t \cdot C^{st}_{rj}$

(60) $\frac{\partial L}{\partial S^t_{rz}} = \beta^t \cdot C^{st}_{rz} - {}^{14}\lambda^t_{rz} - {}^{17}\lambda^t_{rz} = 0$

(60-1) ${}^{14}\lambda^t_{rz} + {}^{17}\lambda^t_{rz} = \beta^t \cdot C^{st}_{rz}$

(61) $\frac{\partial L}{\partial \lambda} = 0$; for all λs.

V. CONCLUSIONS

An attempt has been made to bring together and formalize some of the factors that seem to exist in "Supply and Demand for Energy". After a brief review of the world energy consumption it is shown that there is a broad parallel between economic growth and energy consumption. It is important to explore the relationship as precisely as possible and thus more intensive research is needed to determine:

a. The impact on energy consumption of technological efficiencies in consuming countries.

b. The "average" nation-wide levels of energy consumption required for given level of GNP.

c. The effect of above mentioned topics studied in terms of specific energy sources. In addition, the substitutability of one energy source for another in satisfying requirement should be considered.

d. A more specific effect of relative prices of resources on energy consumption.

e. Impacts of environmental controls and pollution abatement upon the volume of energy demanded and upon shifts to substitute energy sources.

f. Impacts of economic and financial policies such as balance of payments or balance of trade considerations upon the volume of energy consumption.

By introducing the above mentioned considerations in the

empirical analysis of past data, a greater insight might be gained into future projection of energy requirements.

In the analytical section of this study the Cummings and Whipple model was extended by posting a relationship between extraction, storage, import, exploration and market prices of resources and relative cost factors.

The model does not include social values and environmental considerations.

The model would allow for price regulatory policy decisions, and considers both supplies and demands of resources, and their interactions is achieved through the dynamic market mechanism. It is designed for repeated use and considers the technological impact and conversion of capital stocks.

The model is disaggregated by regions, covers all energy forms and takes into account the interfuel substitution brought about by price changes.

It is believed that expansion of the model to cover the following additional areas, would make it more useful in conjunctive management.

1) Inclusion of environmental considerations and pollution abatement.

2) Inclusion of specific functional forms for extraction, import, investment, etc. These could be obtained by statistic estimation.

3) The inclusion of uncertainty in exploration expenditures.

4) Inclusion of transportation problems.

5) Refinement of the model so that a computer simulation might be employed.

APPENDIX A

1. <u>Compound interest and rate of growth</u>

If $\$y_0$ is invested at compound interest at r% per year, compounded yearly then the amount after t years is $\$y$ where:

(1) $$y = y_0(1 + r)^t$$

if the interest is added n times a year, then

(2) $$y = y_0\left(1 + \frac{r}{n}\right)^{nt}$$

Discontinuity is an essential feature of this compound interest problem. The more frequently is interest added, the larger is the amount of a given sum at the end of any period.

Letting n, the number of the frequency of compounding interest to take larger and larger values, it is clear that $\left(1 + \frac{1}{n}\right)^n$ tends to a definite limit and it can be shown that

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n ; \quad e = 2.71828$$

Returning to the case of compound interest

$$y = y_0\left(1 + \frac{r}{n}\right)^{nt} = y_0\left\{\left(1 + \frac{r}{n}\right)^{\frac{n}{r}}\right\}^{rt} = y_0\left\{\left(1 + \frac{1}{m}\right)^m\right\}^{rt}$$

for $n\to\infty$; $\left(1 + \frac{1}{m}\right)^m = e$ then

(3) $$y = y_0e^{rt}$$

We have now derived a concept for continuous interest.

Note: This model represents a constant percent rate of growth (interest rate). It is clear that the percent rate

of growth at time t is $r(t) = \frac{\frac{dy}{dt}}{y}(100)$ where $\frac{dy}{dt}$ represents the rate of change.

2. Present Values

a. If the interest is added once yearly at r% then y is the present value of y available after t years; from Eq. (1):

$$y_0 = \frac{y}{(1+r)^t} \tag{4}$$

b. If interest is added n times a year at r% from Eq. (2):

$$y_0 = \frac{y}{(1+\frac{r}{n})^{nt}} \tag{5}$$

c. Finally, if interest is added continuously at r% from Eq. (3) it implies

$$y_0 = \frac{y}{e^{rt}} \quad \text{or} \quad y_0 = y \cdot e^{-rt} \tag{6}$$

APPENDIX B

Calculus of variations - Euler's Equation.[1]

If the extreme values of the integral

$$u = \int_{t_0}^{t_1} f(t, q(t), \frac{dq}{dt})dt$$

are required for all possible variation in the function $q=q(t)$ such that $q(t_0)=q_0$ and $q(t_1)=q_1$ where (t_0,q_0) and (t_1,q_1) are fixed points. The function

$$f(t) \equiv f(t,q,\frac{dq}{dt}) \equiv f\{t,q(t),\dot{q}(t)\}$$

which gives variable u on integration, depends on the variable t, on the variable function $q=q(t)$ and on the first derivative $\dot{q}(t)$. This is a function of t given in the function of functions form. In solving the problems, the function $q(t)$ is taking in the form $q=q(t;\alpha,\beta,\gamma,\ldots)$ where $q(t;\alpha,\beta,\gamma,\ldots)$ is assumed to be a fixed function (with a continuous derivative) and where $\alpha,\beta,\gamma,\ldots$ are parameters. Given differential increments $\delta\alpha$, $\delta\beta$, $\delta\gamma$, ... to the parameters, we derive the corresponding variations δq and $\delta\dot{q}$ in the function q and its derivative $\dot{q} = \frac{dq}{dt}$:

(1) $$\delta q = \frac{\partial q}{\partial \alpha}\delta\alpha + \frac{\partial q}{\partial \beta}\delta\beta + \frac{\partial q}{\partial \gamma}\delta\gamma + \ldots.$$ and

(2) $$\delta q = \delta(\frac{dq}{dt}) = \frac{\partial}{\partial\alpha}(\frac{dq}{dt})\delta\alpha + \frac{\partial}{\partial\beta}(\frac{dq}{dt})\delta\beta + \frac{\partial}{\partial\gamma}(\frac{dq}{dt})\delta\gamma + \ldots.$$

$$= \frac{d}{dt}(\frac{\partial q}{\partial\alpha})\delta\alpha + \frac{d}{dt}(\frac{\partial q}{\partial\beta})\delta\beta + \frac{d}{dt}(\frac{\partial q}{\partial\gamma})\delta\gamma + \ldots$$

$$= \frac{d}{dt}(\frac{\partial q}{\partial\alpha}\delta\alpha + \frac{\partial q}{\partial\beta}\delta\beta + \frac{\partial q}{\partial\gamma}\delta\gamma + \ldots) = \frac{d}{dt}(\delta q)$$

---

[1] Reference 7, Page 521.

The symbol "δ" is used in order to distinguish parametric differentials from "d" deferring to variation in variable t. The function $f(t,q,\frac{dq}{dt})$ and the integral u can now be considered as dependent on the parameters $\alpha,\beta,\gamma,\ldots$ and the variation in their values are obtained as:

$$\delta f = \frac{\partial f}{\partial q}\delta q + \frac{\partial f}{\partial \dot{q}}\delta \dot{q}$$

from Eq. (2) results

$$(3) \qquad \delta f = \frac{\partial f}{\partial q}\delta q + \frac{\partial f}{\partial \dot{q}}\frac{d}{dt}(\delta q)$$

and

$$\delta u = \int_{t_0}^{t_1} f(t,q,\dot{q})dt = \int_{t_0}^{t_1} \delta f\, dt$$

From Eq. (3)

$$(4) \qquad \delta u = \int_{t_0}^{t_1} (\frac{\partial f}{\partial q}\delta q)dt + \int_{t_0}^{t_1} \{\frac{\partial f}{\partial \dot{q}}\frac{d}{dt}(\delta q)\}dt$$

and finally be appropriate substitution and rearrangement. The expression for variation in u becomes:

$$\delta u = [\frac{\partial f}{\partial \dot{q}}\delta q]_{t_0}^{t_1} + \int_{t_0}^{t_1} \{\frac{\partial f}{\partial q} - \frac{d}{dt}(\frac{\partial f}{\partial \dot{q}})\}\delta q\, dt.$$

Since by boundary conditions, the curve $q=q(t)$ always passes through two fixed points $t=t_0$ and $t=t_1$, it follows that the $\delta q=0$ at these points. Hence

$$[\frac{\partial f}{\partial \dot{q}}\delta q]_{t_0}^{t_1} = 0 \qquad \text{and so}$$

$$(5) \qquad \delta u = \int_{t_0}^{t_1} \{\frac{\partial f}{\partial q} - \frac{d}{dt}(\frac{\partial f}{\partial \dot{q}})\}\delta q\, dt\ .$$

The necessary condition for the u to have an extreme value for variation in the function $q=q(t)$ as obtained by varying the parameters $\alpha,\beta,\gamma,\ldots$ is that $\delta u=0$ for all values of $\delta\alpha$, $\delta\beta$, $\delta\gamma$, ..., i.e. for all values of δq from Eq. (5). This is only true if

$$(6) \qquad \frac{\partial f}{\partial q} - \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{q}}\right) = 0$$

This result, known as Euler's equation, determines that function $q=q(t)$ which maximizes or minimizes the value of u. It is a differential equation which must be solved to give the function $q=q(t)$ sought. Euler's equation, however, is only a necessary condition for extreme values of u. For distinguishing between maximum and minimum values, a general criterion is not readily obtainable, in simple practical case, it can usually be judged from general reasoning.

APPENDIX C

<u>OECD Countries used in the Analysis</u>

1. Canada
2. United States
3. Japan
4. Australia ***
5. Austria
6. Belgium
7. Denmark *
8. Finland *
9. France
10. Germany
11. Greece *
12. Iceland **
13. Ireland *
14. Italy
15. Netherlands
16. Norway *
17. Portugal *
18. Spain
19. Sweden *
20. Switzerland *
21. Turkey *
22. United Kingdom

* Not used in Natural Gas Analysis.

** Used only in Aviation Fuel, Motor Gasoline, Kerosine and Gas Oil Analysis.

*** Used only in Energy/Capita Analysis except for the years 1966, 1967.

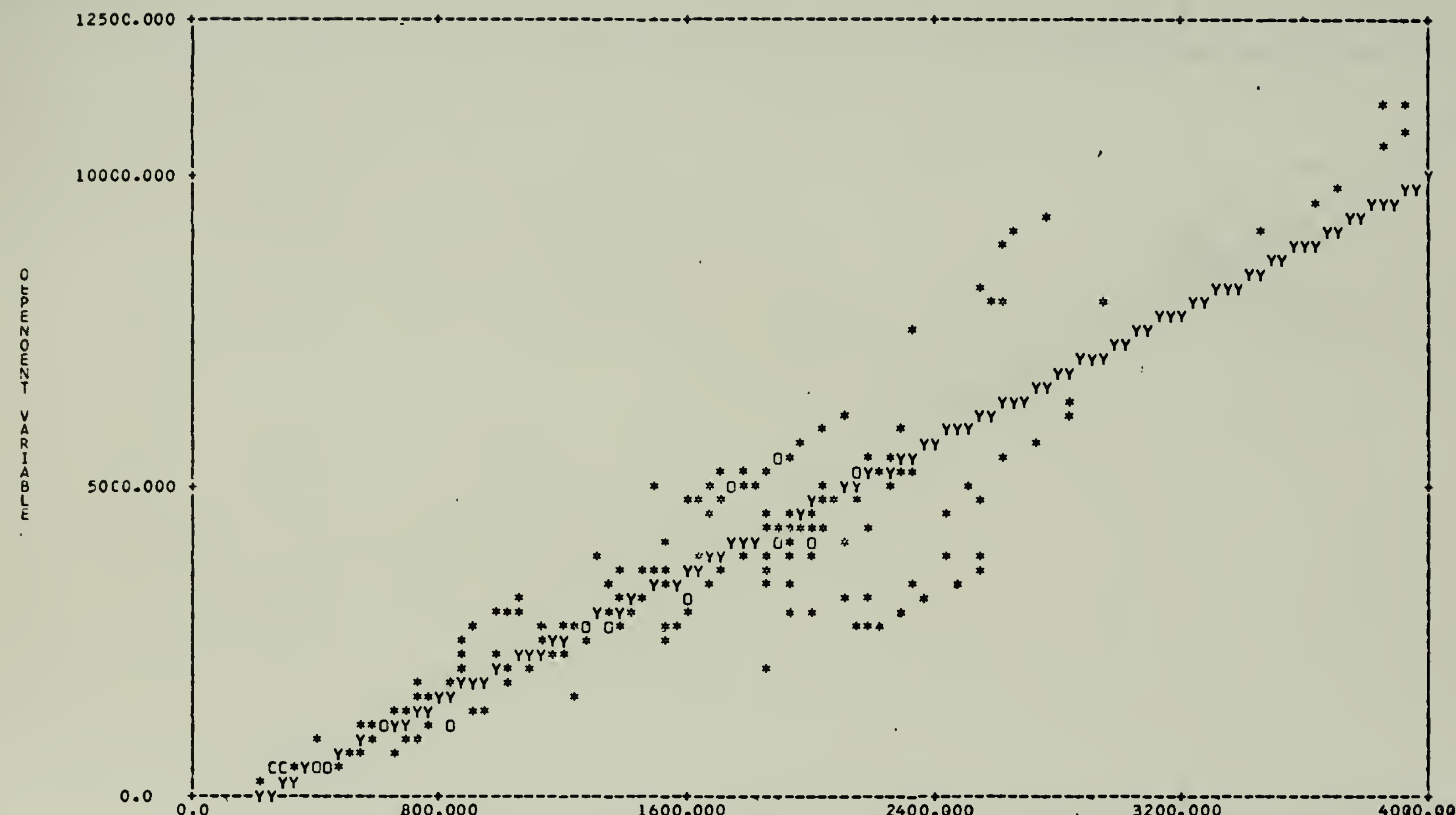

Figure 1. ENERGY/CAPITA AND GNP/CAPITA

Linear Model

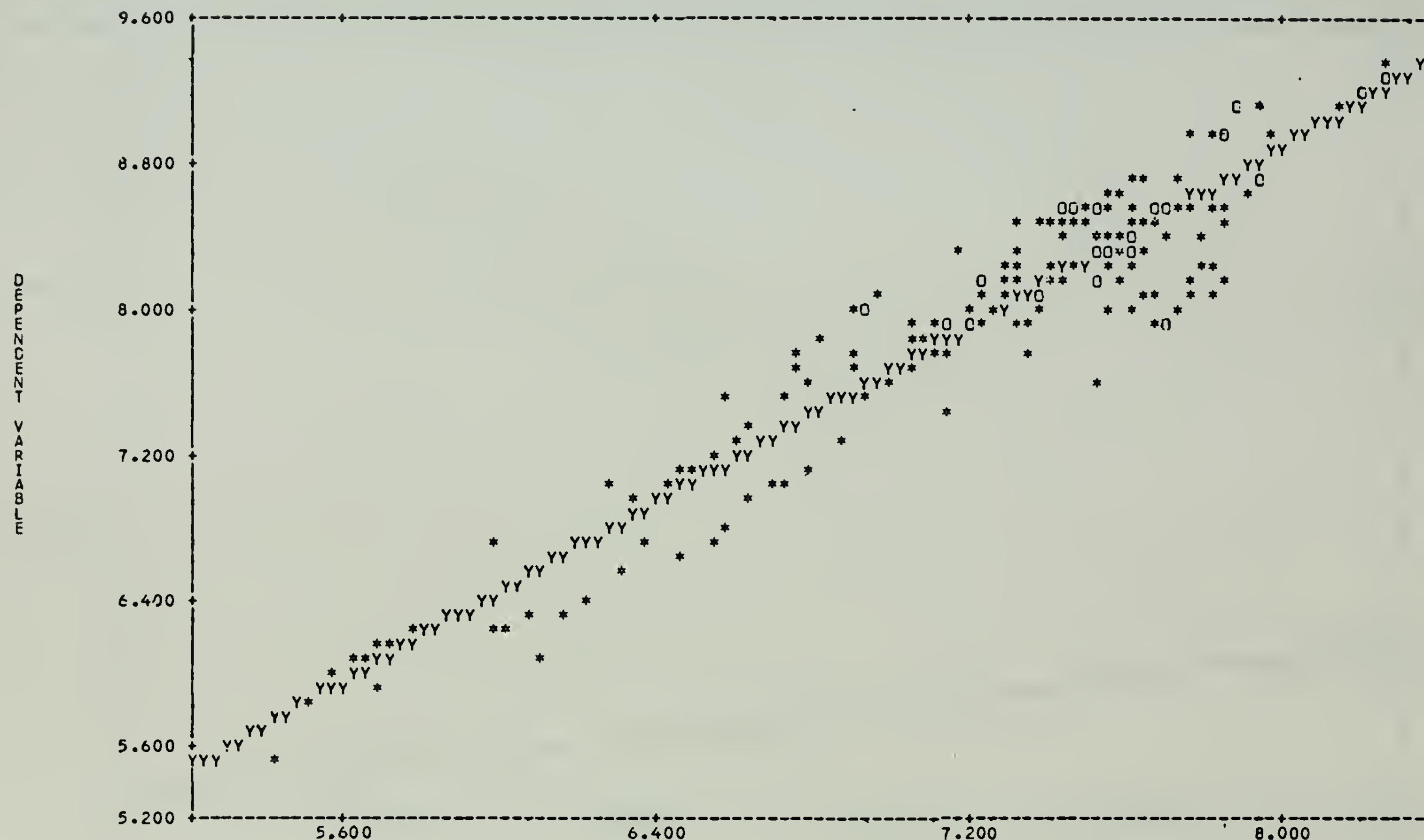

Figure 2. ENERGY/CAPITA AND GNP/CAPITA

Constant Elasticity Model

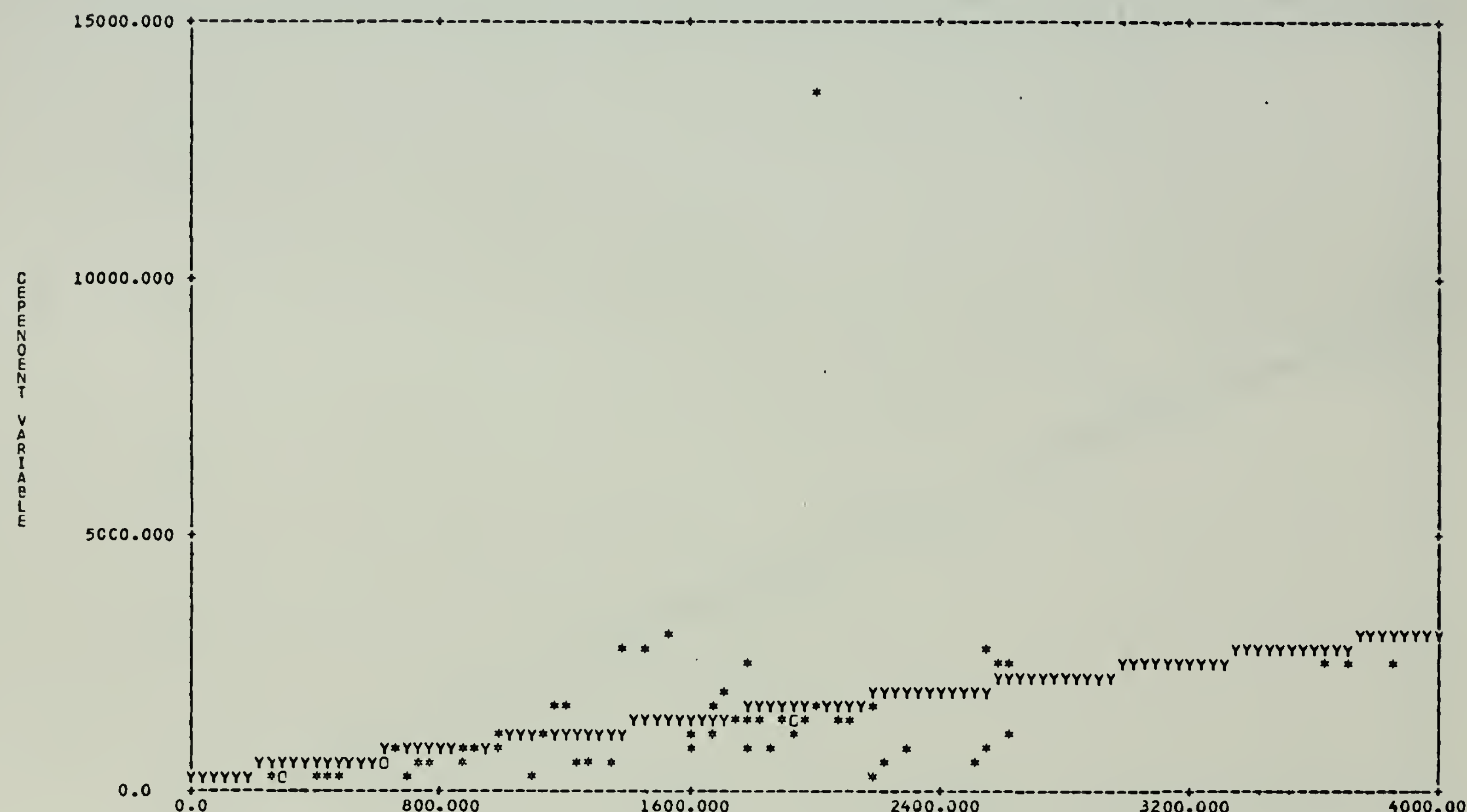

Figure 3. CRUDE PETROLEUM/CAPITA AND GNP/CAPITA

Linear Model

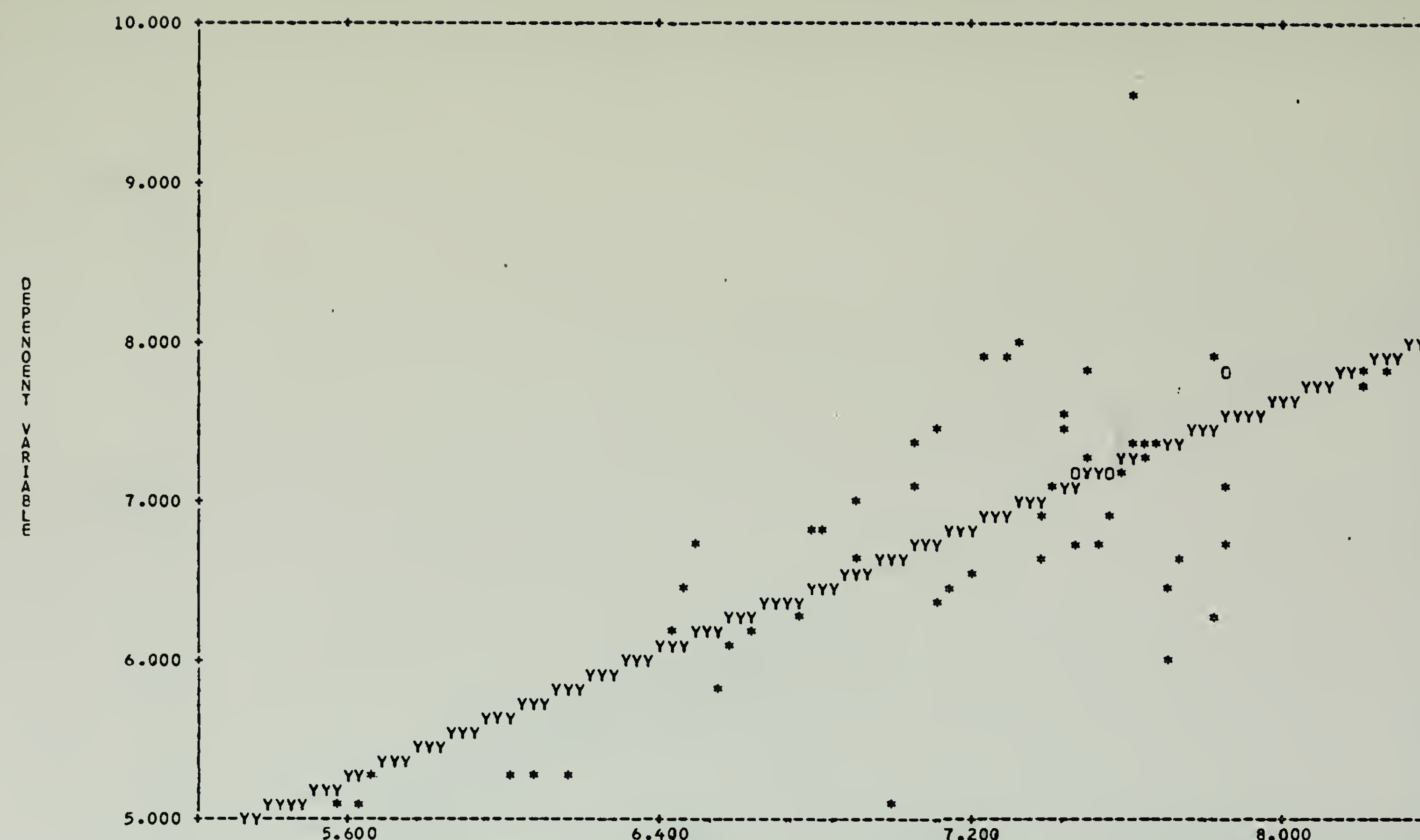

Figure 4. CRUDE PETROLEUM/CAPITA AND GNP/CAPITA

Constant Elasticity Model

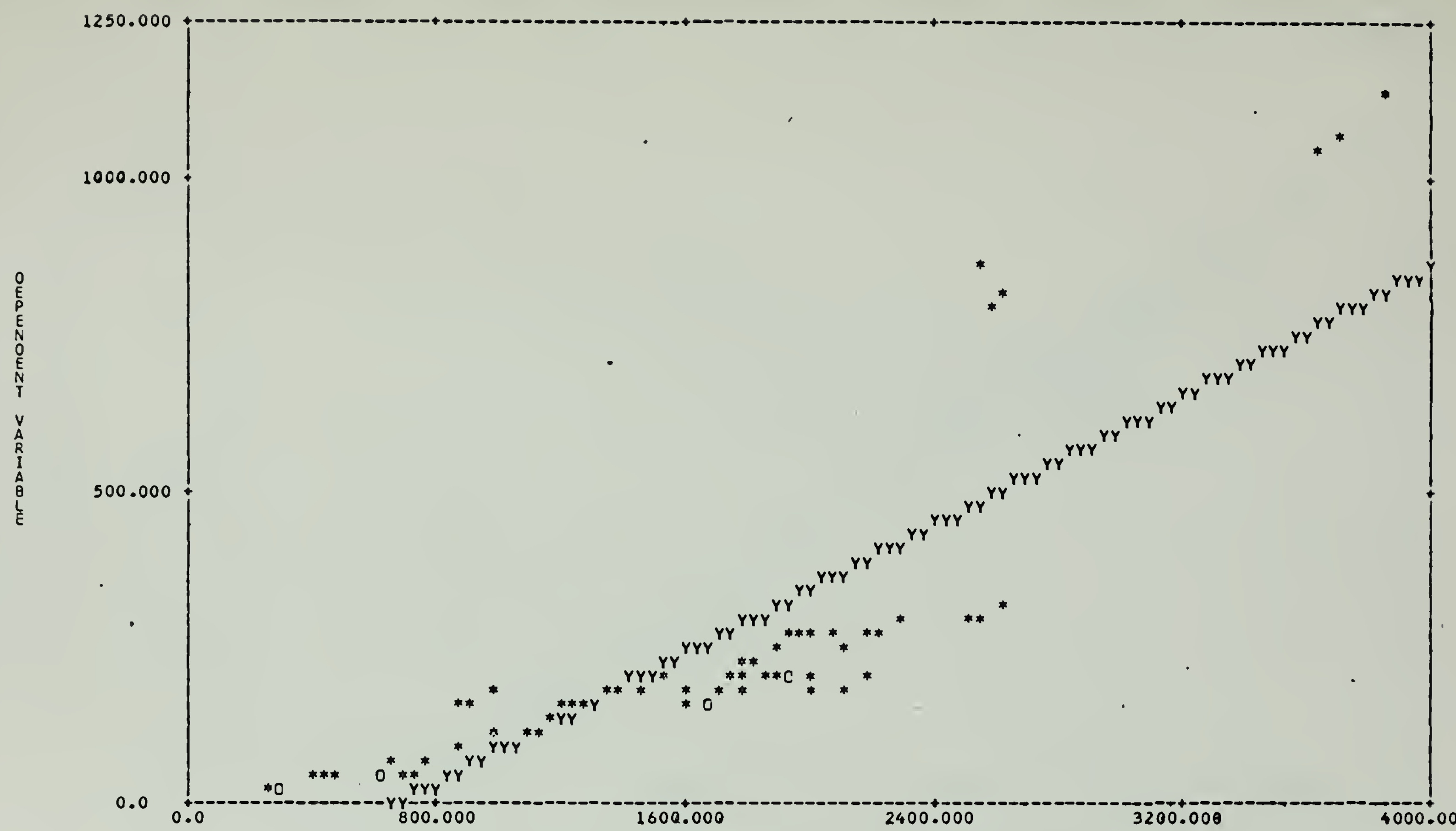

Figure 5. MOTOR GASOLINE/CAPITA AND GNP/CAPITA

Linear Model

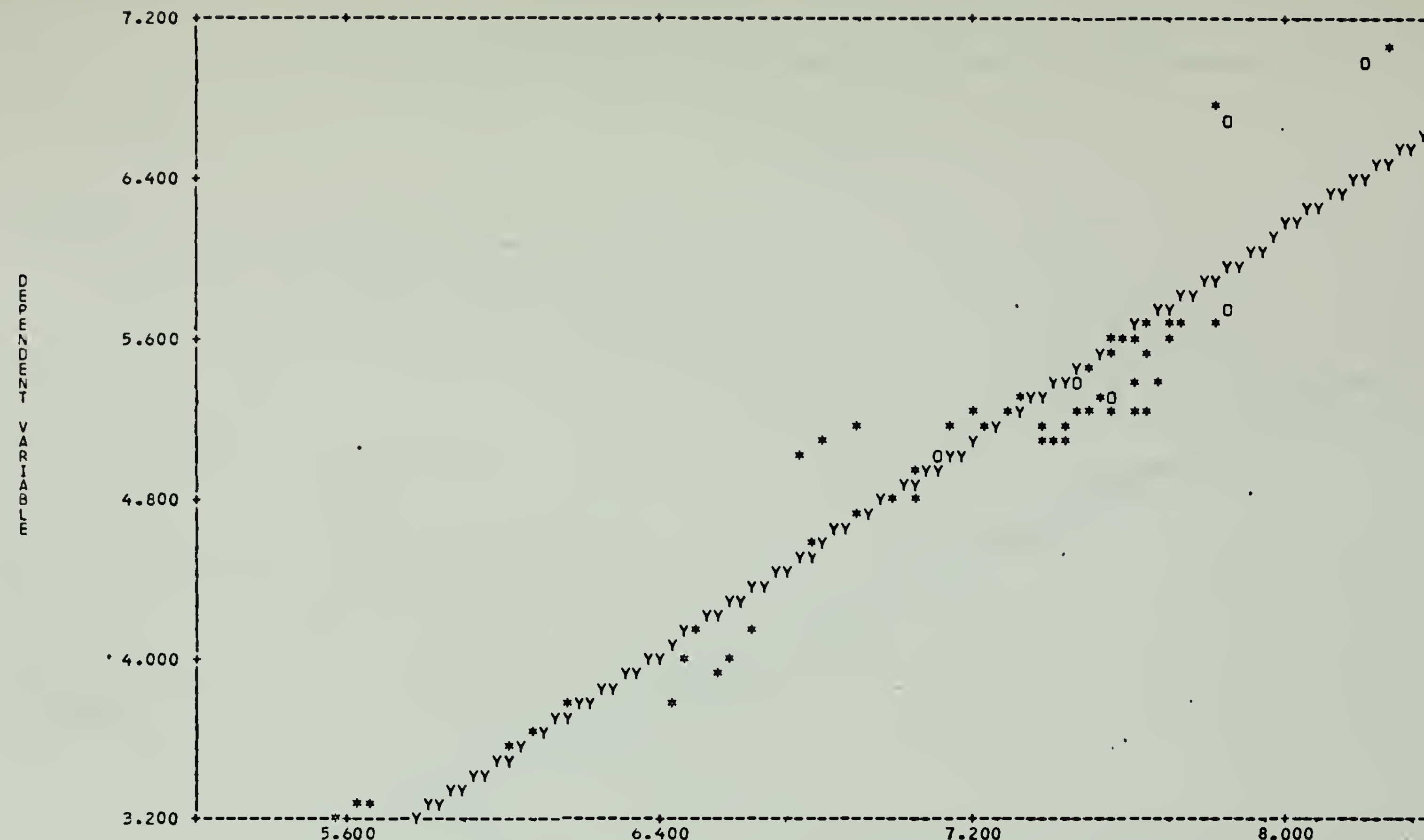

Figure 6. MOTOR GASOLINE/CAPITA AND GNP/CAPITA

Constant Elasticity Model

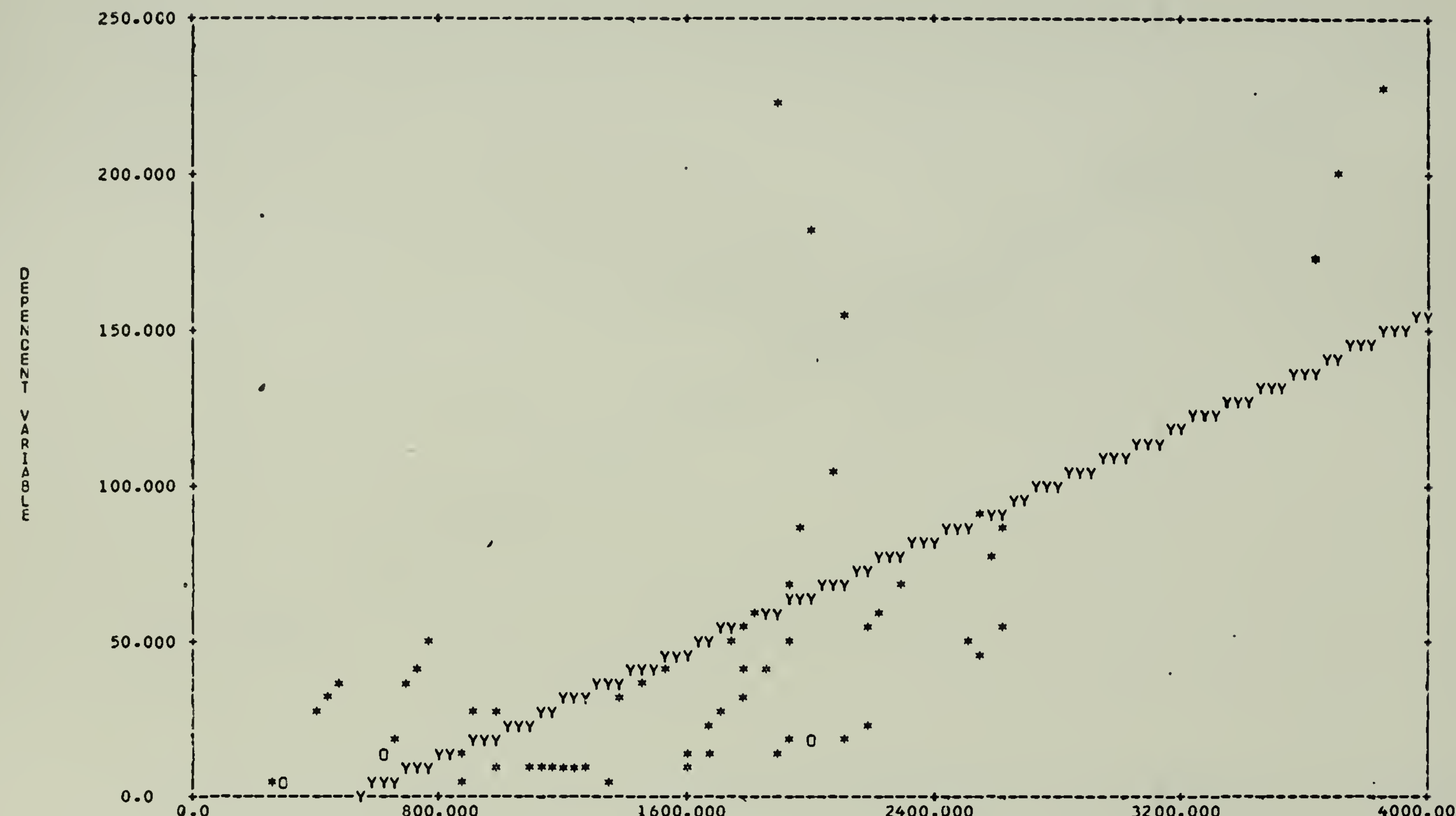

Figure 7. AVIATION FUEL/CAPITA AND GNP/CAPITA

Linear Model

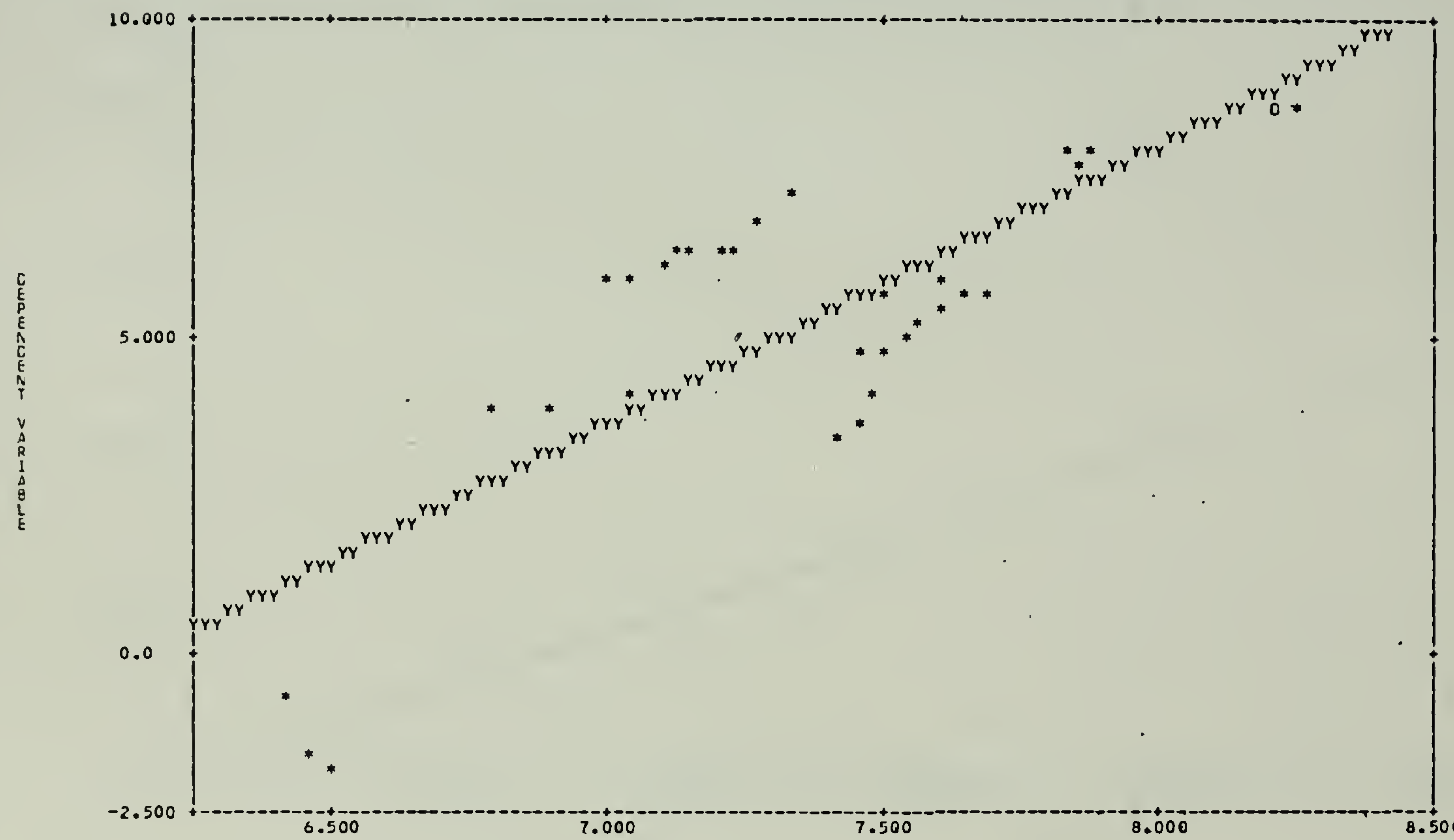

Figure 8. AVIATION FUEL/CAPITA AND GNP/CAPITA

Constant Elasticity Model

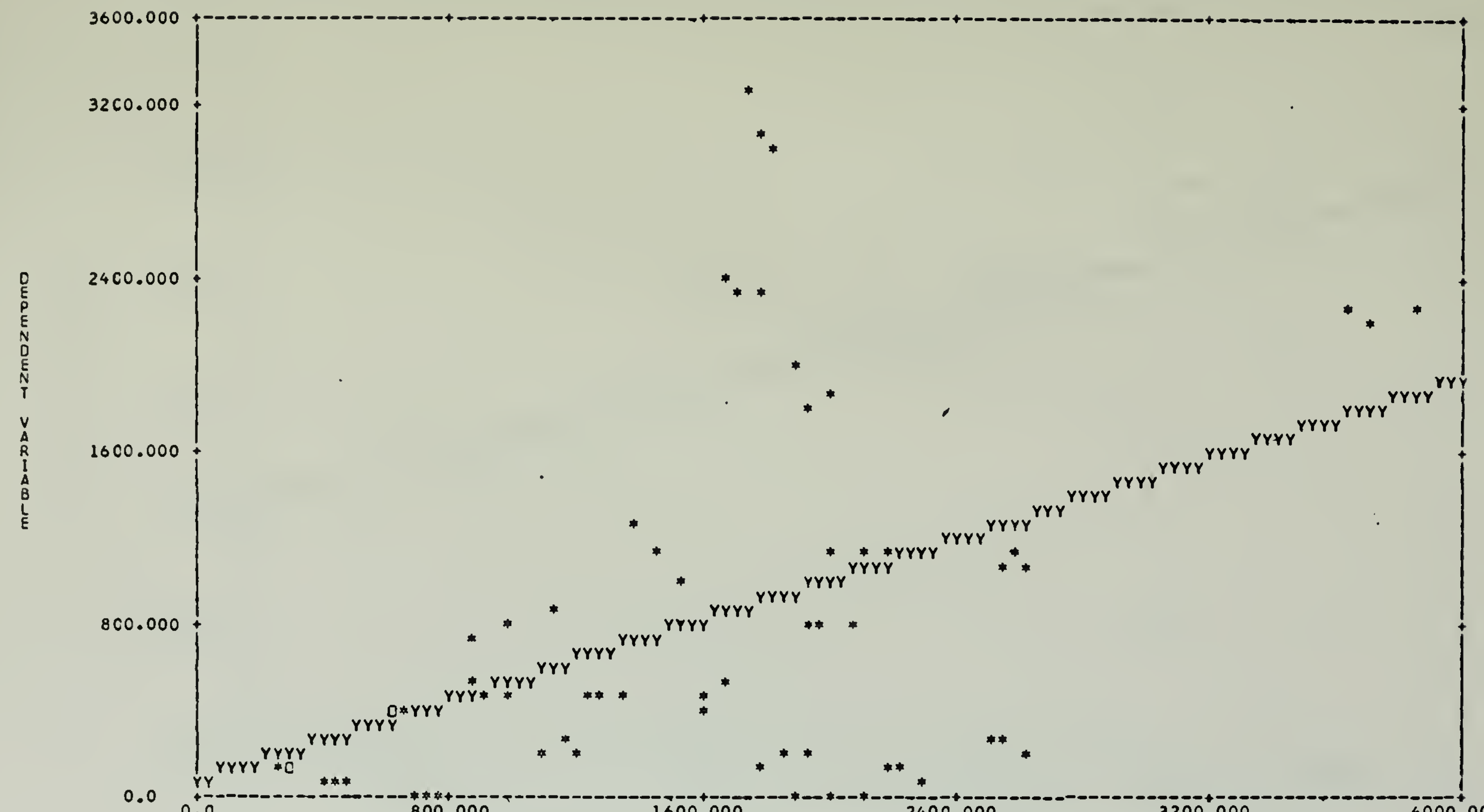

Figure 9. HARD COAL/CAPITA AND GNP/CAPITA

Linear Model

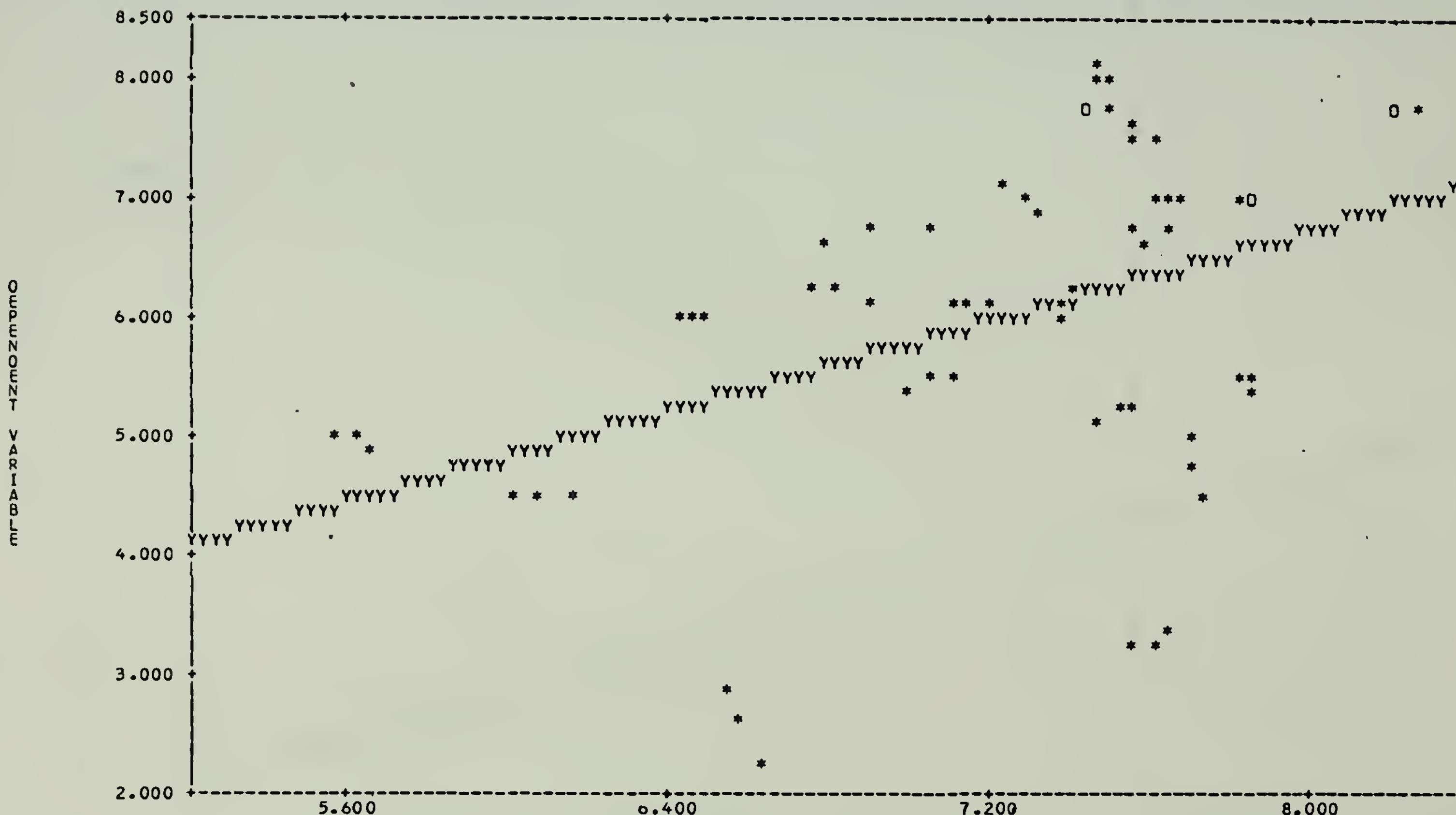

Figure 10. HARD COAL/CAPITA AND GNP/CAPITA

Constant Elasticity Model

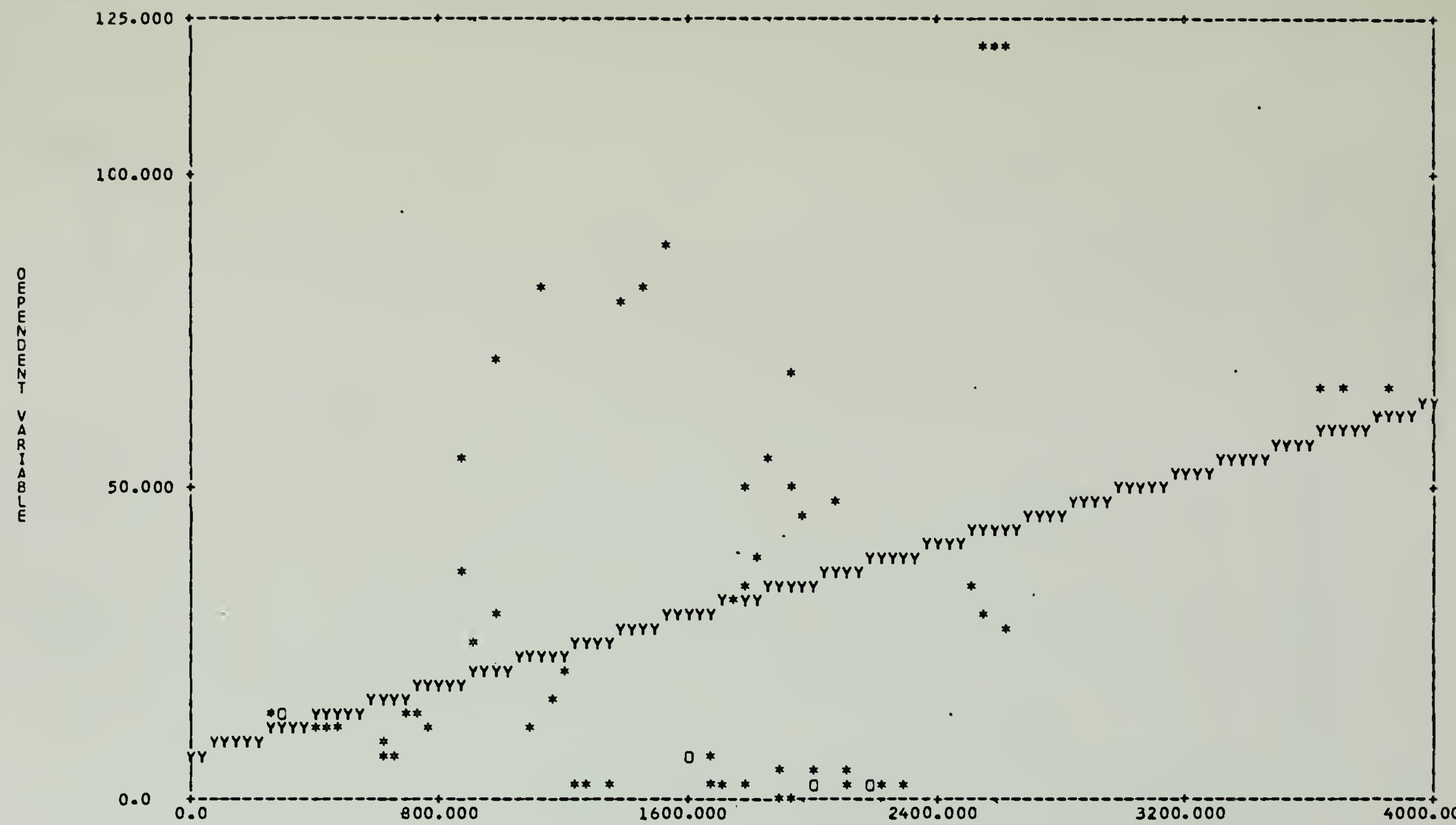

Figure 11. KEROSINE/CAPITA AND GNP/CAPITA

Linear Model

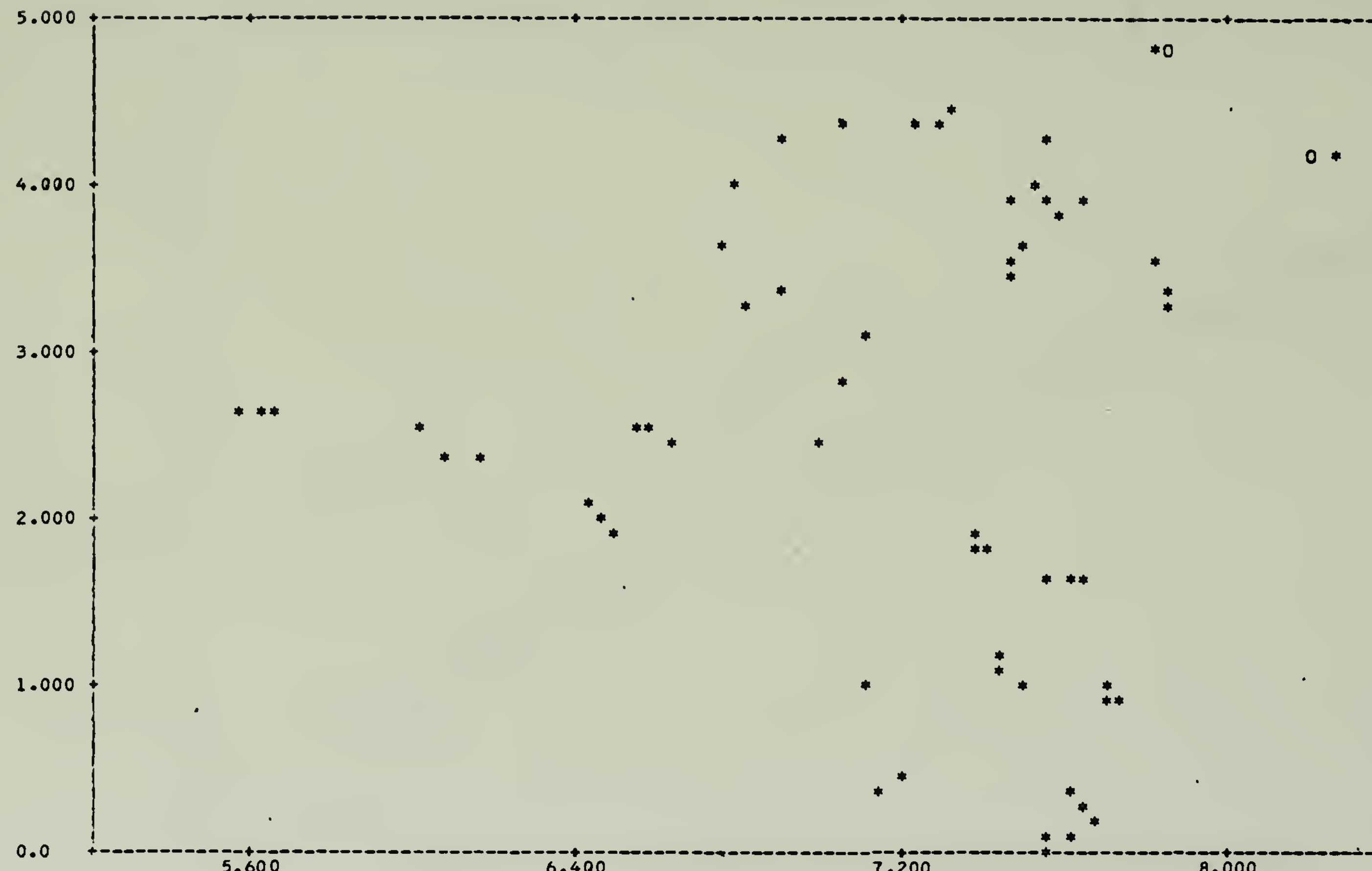

Figure 12. KEROSINE/CAPITA AND GNP/CAPITA

Constant Elasticity Model

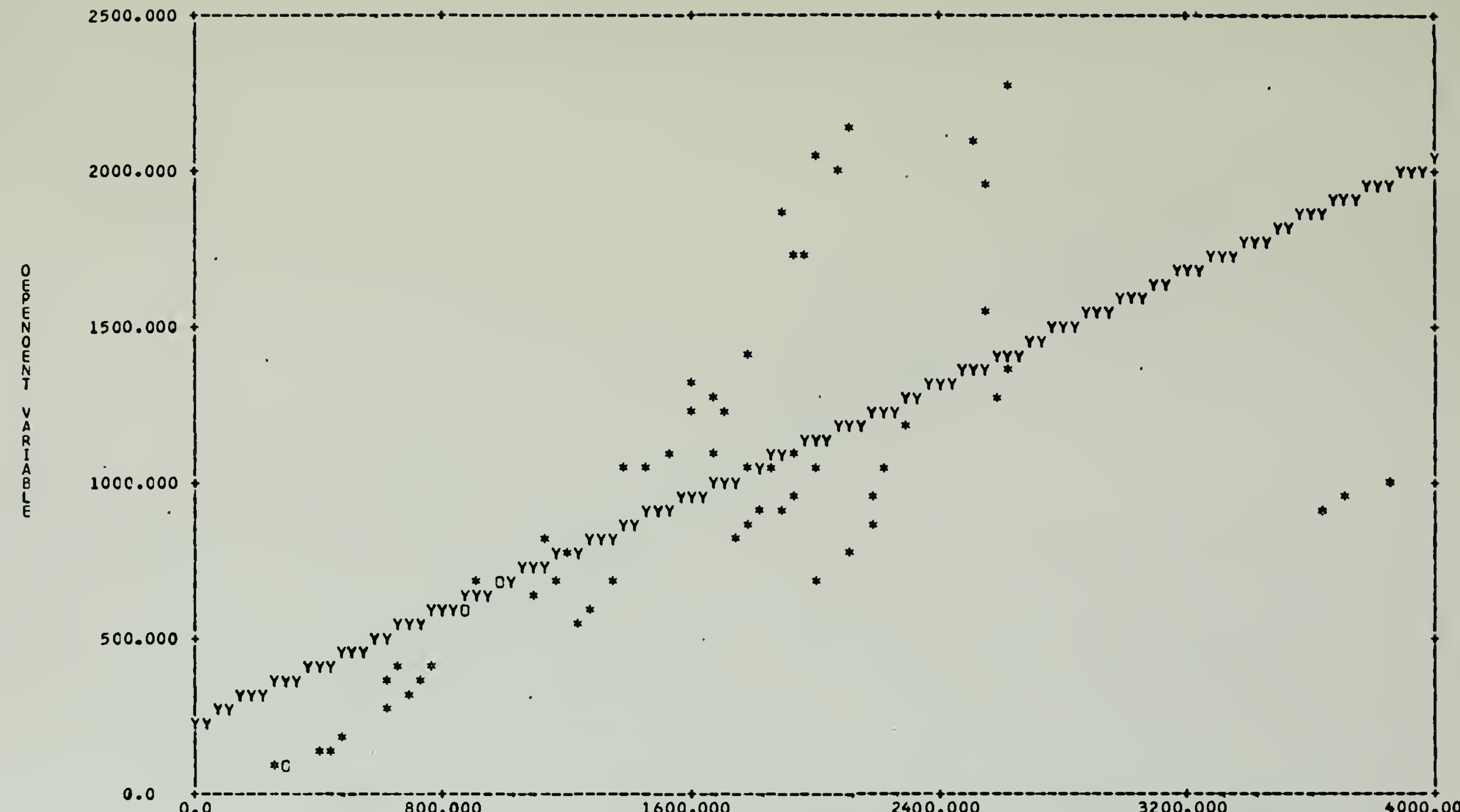

Figure 13. GAS OIL/CAPITA AND GNP/CAPITA

Linear Model

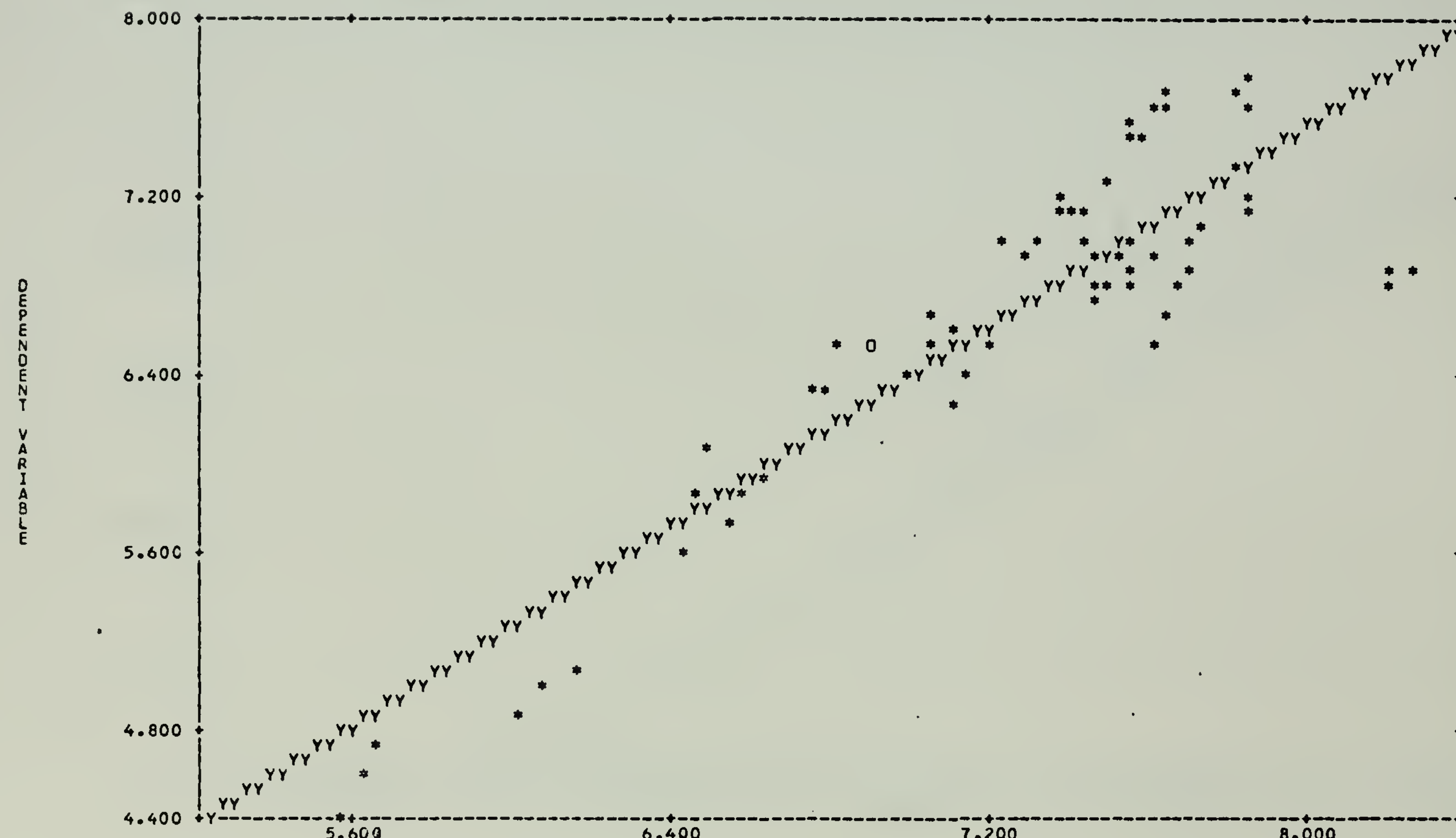

Figure 14. GAS OIL/CAPITA AND GNP/CAPITA

Constant Elasticity Model

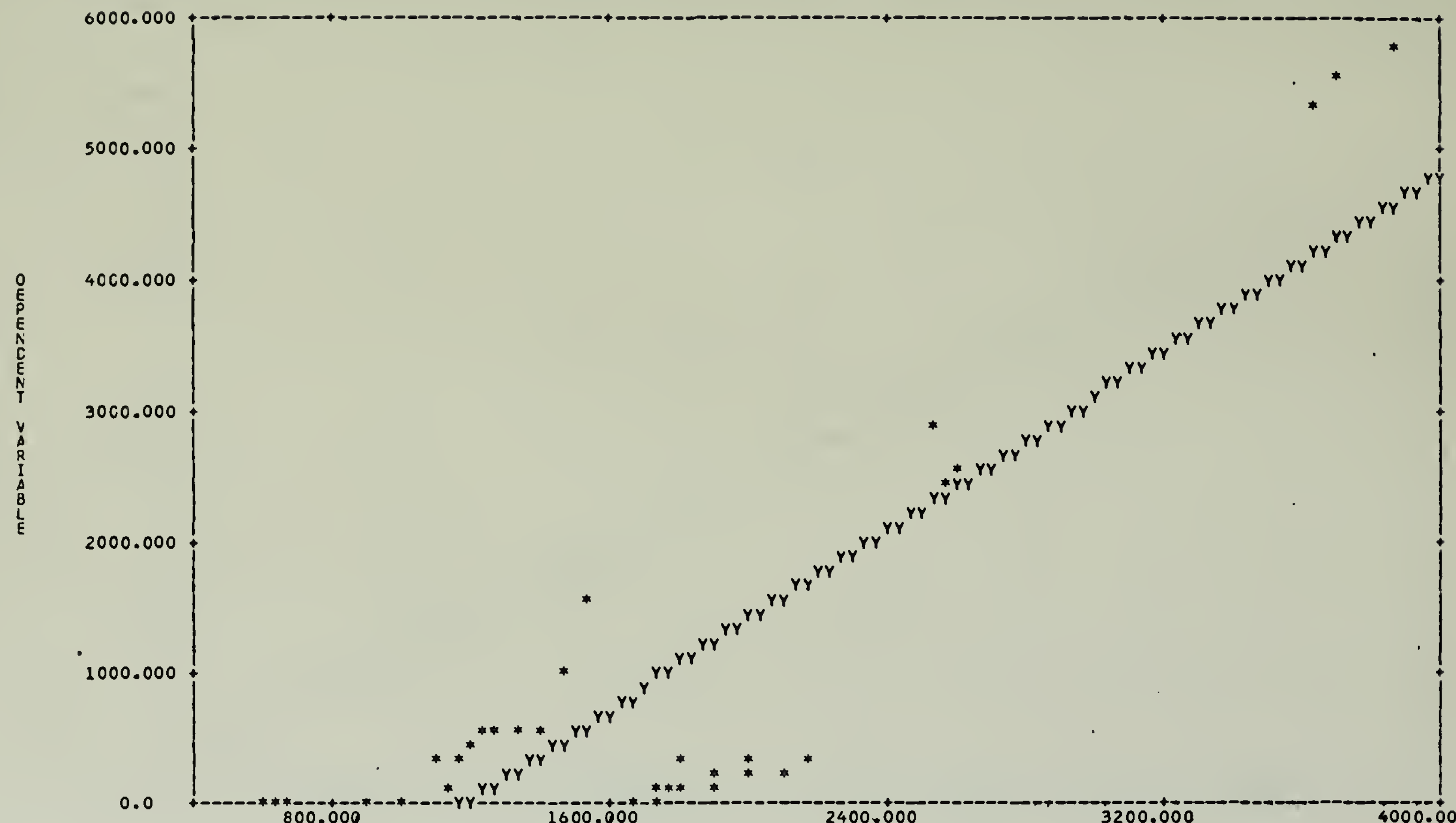

Figure 15. NATURAL GAS/CAPITA AND GNP/CAPITA

Linear Model

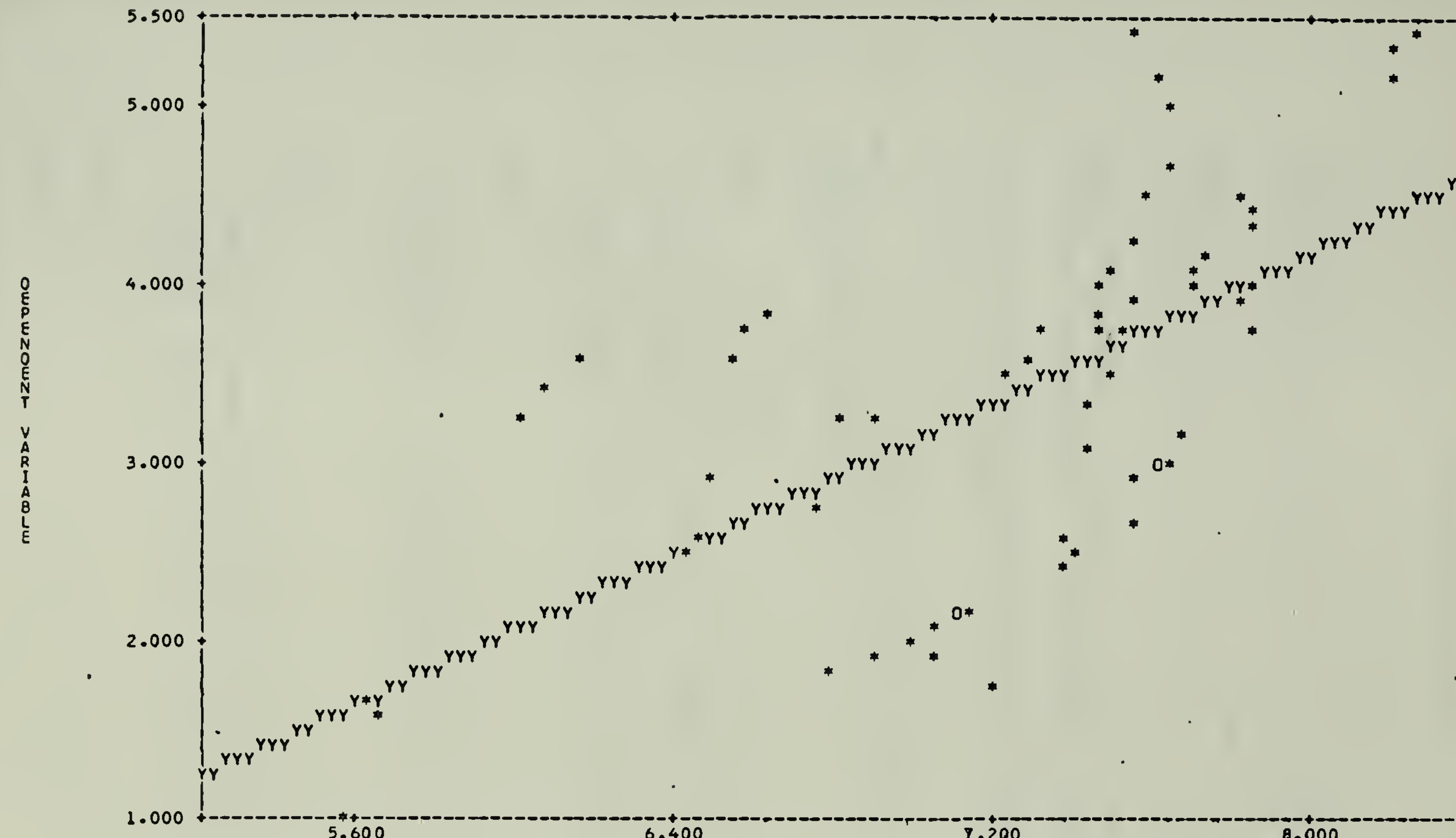

Figure 16. NATURAL GAS/CAPITA AND GNP/CAPITA

Constant Elasticity Model

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